



Introduction to Problem Solving

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Four Principles

How to Solve It suggests the following steps when solving a Mathematical Problem:

- Understand the problem
- Make a plan
- Carry out the plan
- Look back on your work. How could it be better?

Second principle: Devise a plan

- 1. Use Diagrams / Models
- 2. Act it Out
- 3. Use Before & After
- 4. Use Systematic Listing
- 5. Look for Patterns
- 6. Work Backwards
- 7. Use Guess & Check
- 8. Simplify the Problem
- 9. Make Supposition
- 10. Solve Part of the Problem
- 11. Paraphrase the Problem

Danny raises some chickens and rabbits in this little farm. These animals have 15 heads and 40 feet altogether. How many chickens and rabbits does he raise?

Step One: Understand

- How many chickens and rabbits are there altogether?
- How many feet are there altogether?
- How many feet does each chicken have?
- How many feet does each rabbit have?

Problem Solving

Problem → Diagram → Calculation

- *Mathematical concepts or problems sometimes require an illustrative calculation.*
- *Mathematical Concepts can occur in a variety of situations, giving the opportunity to use what is known.*

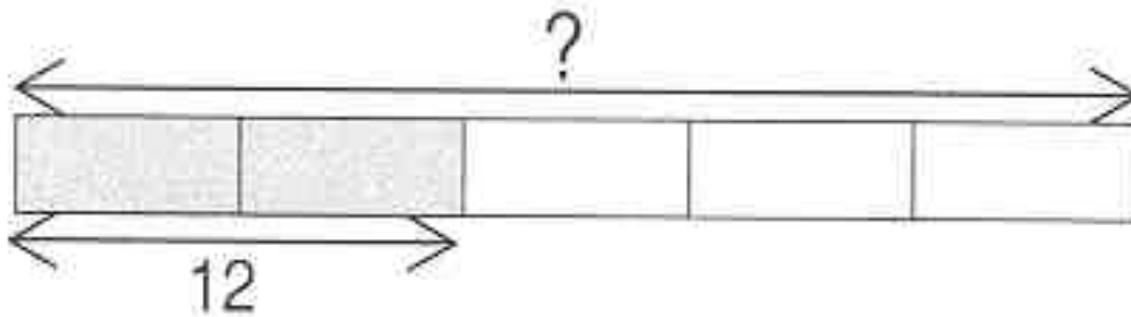
Kathy bought a blouse for three times as much as her scarf cost. The scarf cost half as much as her hat. Her hat cost \$10.00. How much did her blouse cost?

Use of these models to solve problems

$\frac{2}{5}$ of a number equals 12. What is the number?

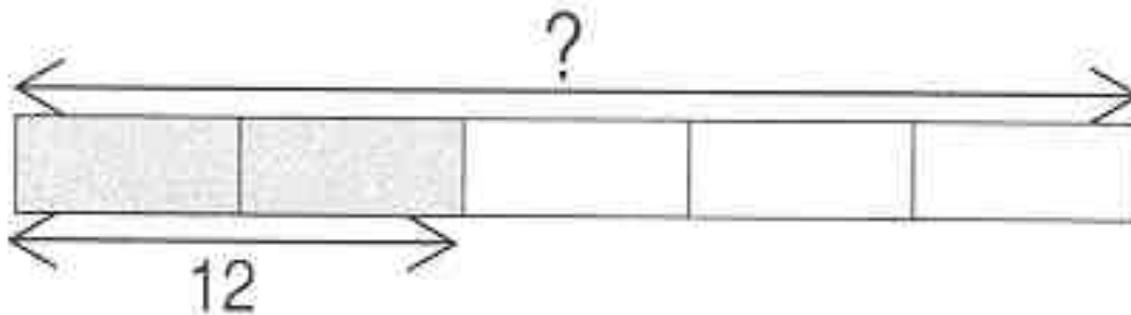
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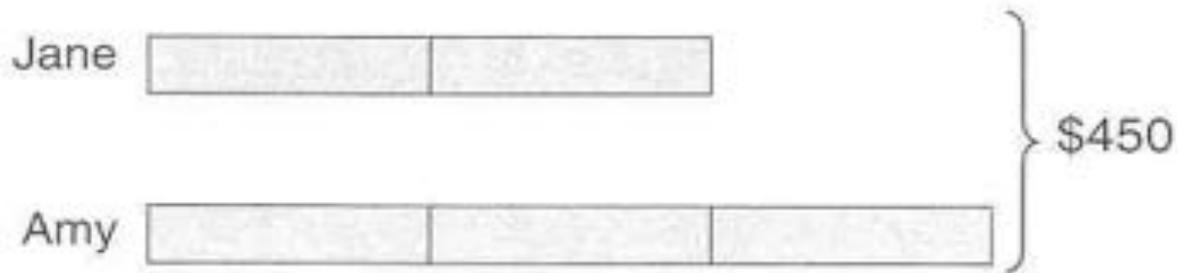
$$2 \text{ units } \longrightarrow 12$$

$$1 \text{ unit } \longrightarrow 12 \div 2 = 6$$

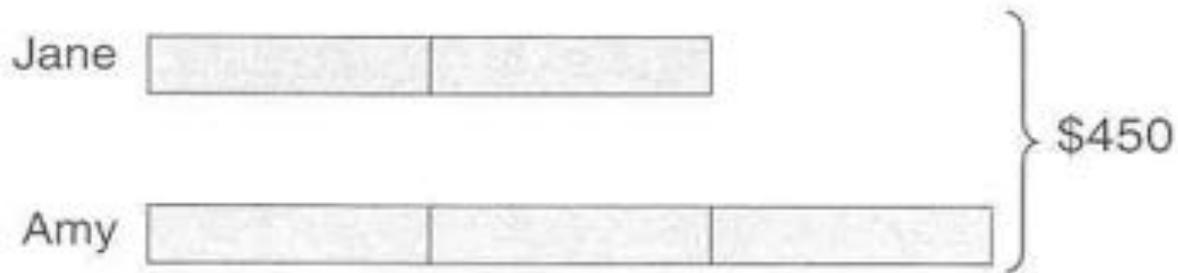
$$5 \text{ units } \longrightarrow 5 \times 6 = 30$$

Jane's savings are two thirds of Amy's savings. Together they save 450 dollars. How much money did Jane save?

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$$5 \text{ units} \longrightarrow \$450$$

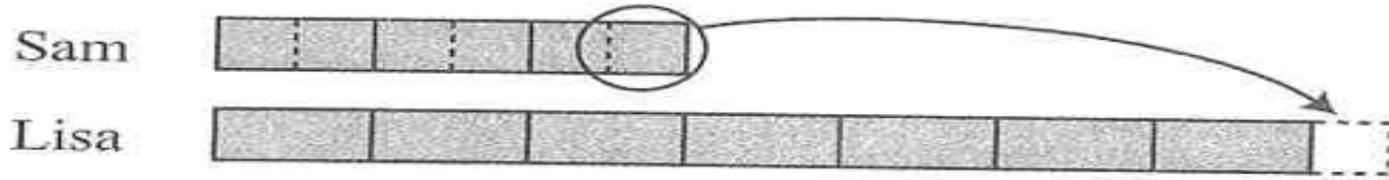
$$1 \text{ unit} \longrightarrow \$450 \div 5 = \$90$$

$$2 \text{ units} \longrightarrow 2 \times \$90 = \$180$$

Sam has $\frac{3}{7}$ the amount of marbles that Lisa has. Sam gives Lisa $\frac{1}{6}$ of his marbles. What will be the new ratio between the number of marbles of Sam and Lisa?

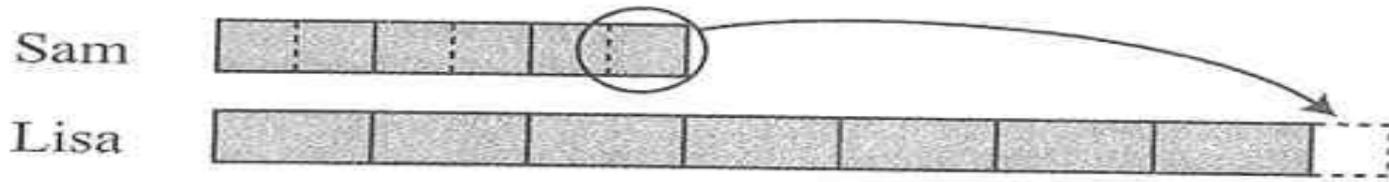
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Before:



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Before:



After

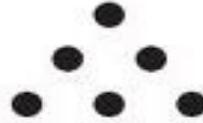




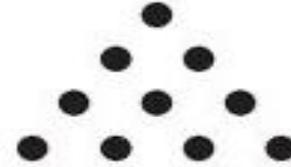
1



3



6

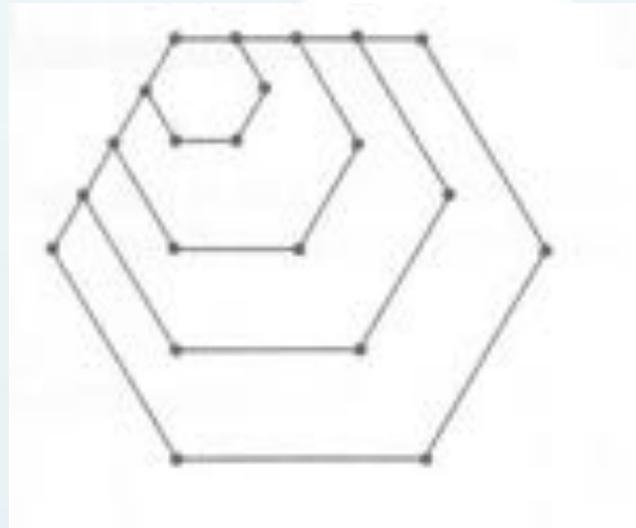


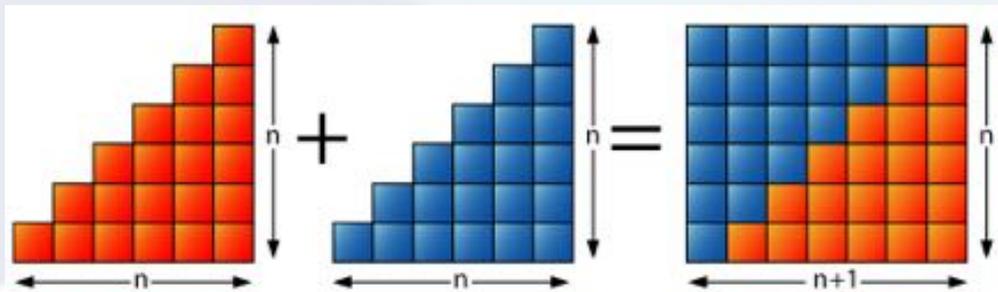
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The first four triangular numbers are 1, 3, 6 and 10.

What is the 10th triangular number?

Robert is bored and started drawing one hexagon, then kept drawing larger and larger hexagons. How many dots he would have altogether after the 8th hexagon?



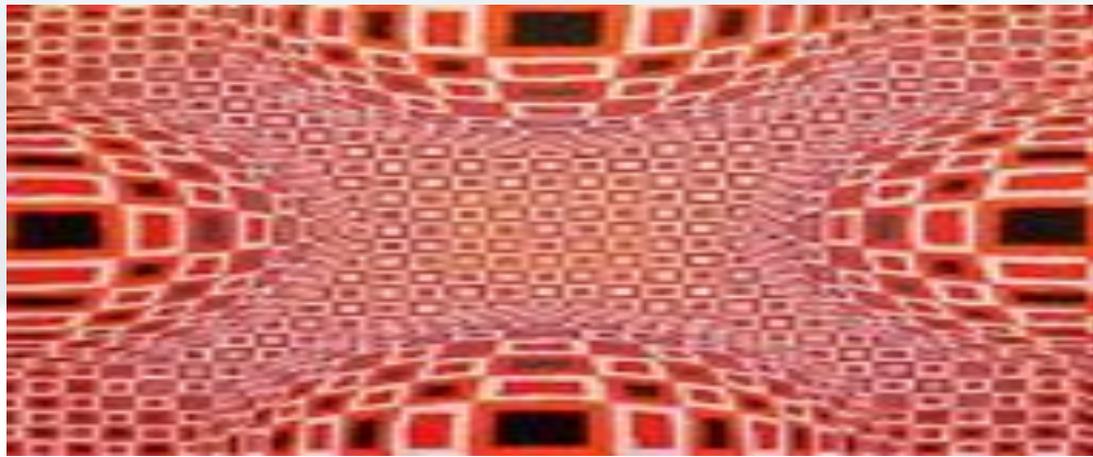


• *Let's find the value of :*

$$1 - 2 + 3 - 4 + 5 - 6 + \dots 99 - 100$$

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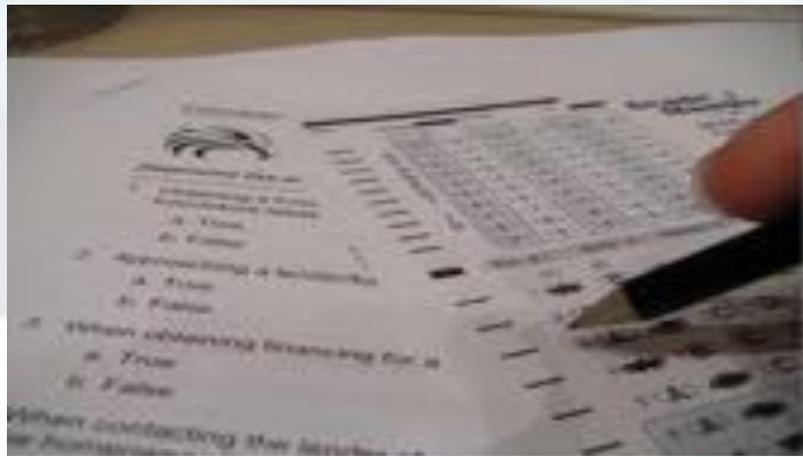
$$1 + 2 + 3 + 4 + 5 + 6 + \dots 99 + 100$$



A piece of wire 52cm long is cut into two parts.

Each part is then bent to form a square. The total area of the two squares is 97 cm².

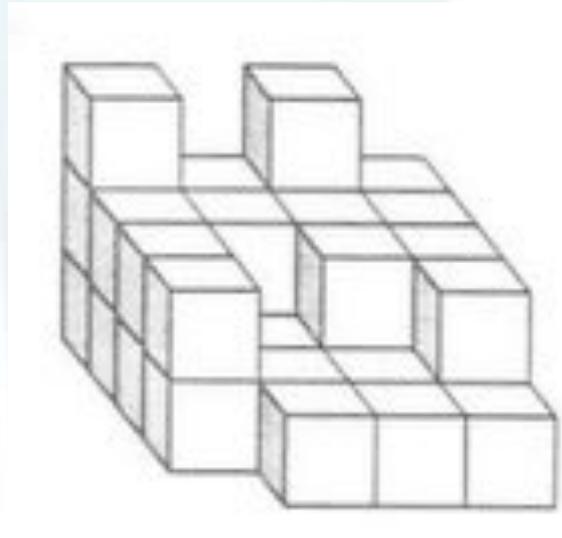
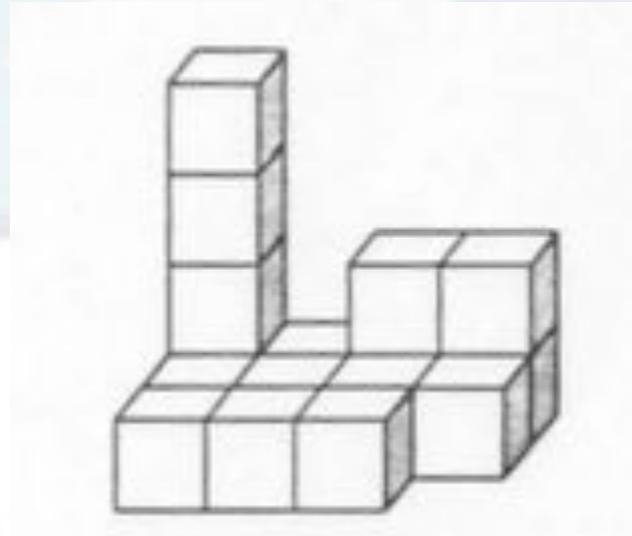
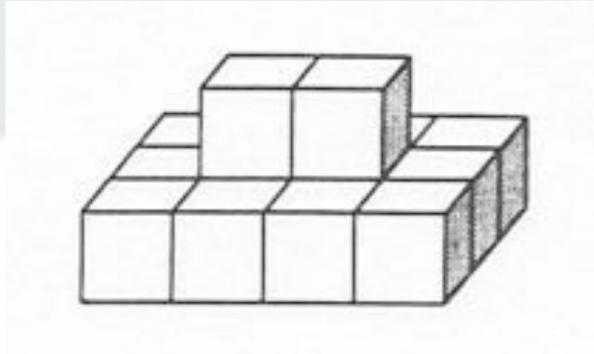
How much longer is a side of the larger square than a side of the smaller square ? (Consider only integers for the lengths of the sides.)



This year in math the teacher will be giving 5 tests, each scored out of 100.

If Heather wishes to get an A+ average (90 or above), what is the lowest possible score she can receive on a test ?

Find the number of cubes in each solid.



Fill in the missing numbers in the boxes.

$$\begin{array}{r} \square 9 \square 3 \\ \times \square \\ \hline 17 \square 58 \end{array}$$

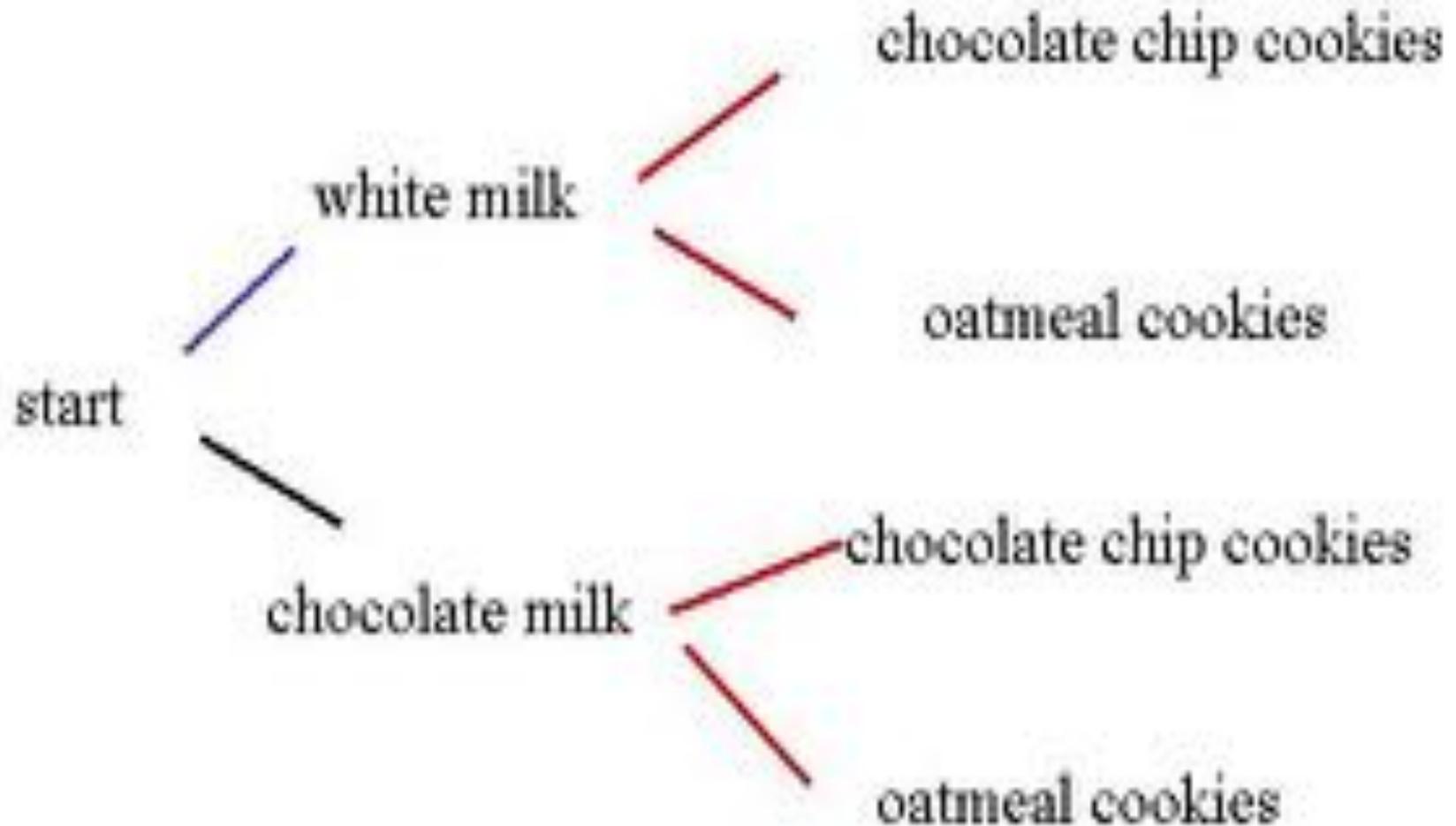
ABCD are four consecutive digits in increasing order. DCBA are the same four digits in decreasing order. The number in the square represents the same four digits in an unknown order. If the sum is 12300. What number is in the square?

$$\begin{array}{rcccc}
 & A & B & C & D \\
 & D & C & B & A \\
 + & \boxed{} & & & \\
 \hline
 1 & 2 & 3 & 0 & 0
 \end{array}$$

Tree Diagram

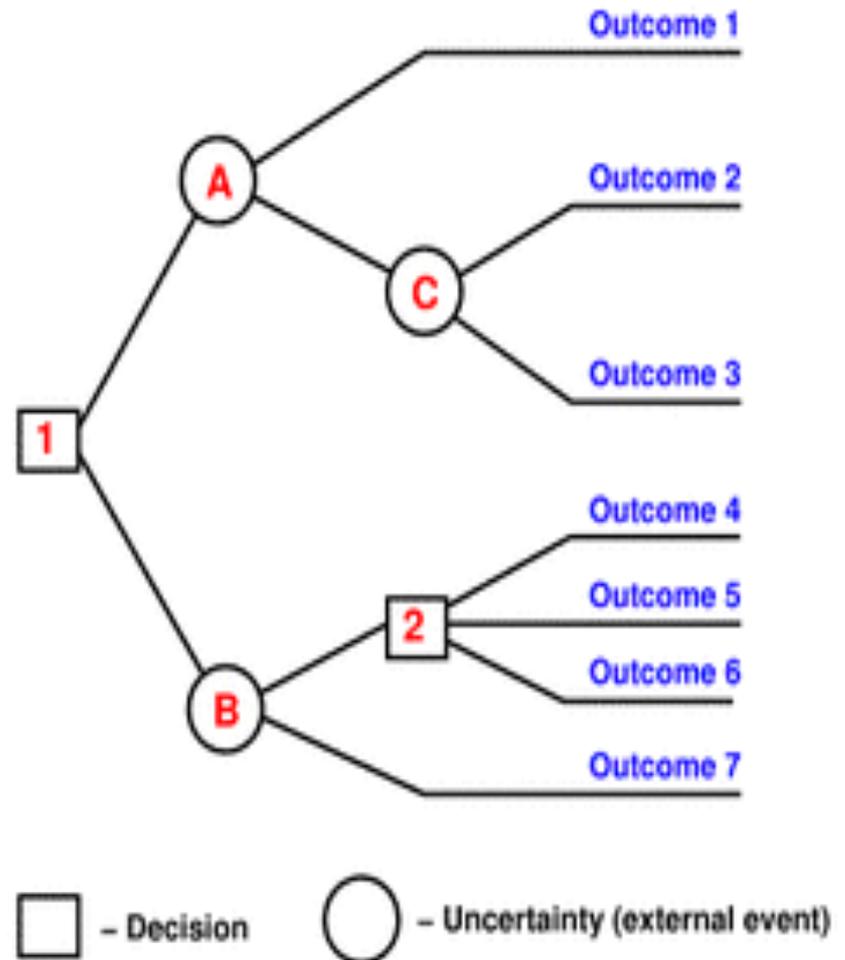
A tree diagram is simply a way of representing a sequence of events. Tree diagrams are particularly useful in probability since they record all possible outcomes in a clear and uncomplicated manner.

Tree Diagram?



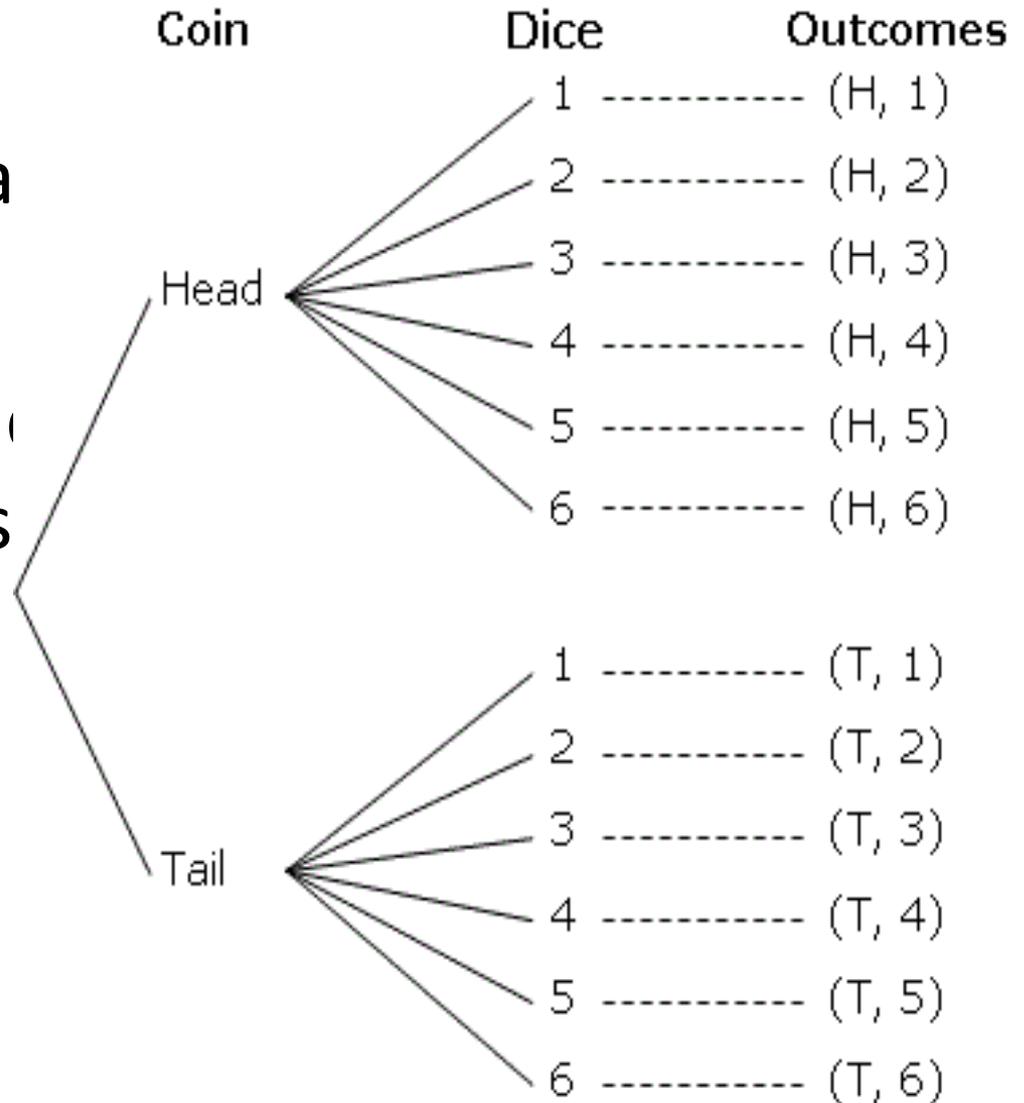
Decision Tree

- Decision Trees help you choose between multiple outcomes/courses you might take. They are very visual and help the user understand the risks and rewards associated with each choice.



Probability problems

- A coin and a dice a
- We can use a tree (possible outcomes



Trigonometry

degrees	radians	$\sin \theta$	$\cos \theta$	$\tan \theta$	$\csc \theta$	$\sec \theta$	$\cot \theta$
0°	0	0	1	0	-	1	-
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
45°	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$
90°	$\frac{\pi}{2}$	1	0	-	1	-	0

- Sine: $\sin(\theta) = \text{Opposite} / \text{Hypotenuse}$
- Cosine: $\cos(\theta) = \text{Adjacent} / \text{Hypotenuse}$
- Tangent: $\tan(\theta) = \text{Opposite} / \text{Adjacent}$

