Introduction

Competition Format

Sprint stage 26 questions
(1 mark for each correct answer)
Page 1: 9 relatively easy questions
Page 2: 9 questions not as easy
Page 3: 8 harder questions

Target stage 12 questions
(2 marks for each correct answer)
3 pages, 4 questions per page
(specified time for each page)
First 2 questions per page are easy
Third question per page is not as easy
Fourth question per page is harder

Maximum total score: 50 points

Count Down stage
Top 10 students per grade compete verbally one on one for trophies and prizes.

Answers to all questions of Sprint and Target stages must be provided in specified space and in specified form:

a. Unless specified otherwise: an integer (positive, zero, or negative)

Examples of acceptable answers:

1, 76, -39, 0, +8

Examples of unacceptable answers:

1.0, 76.1, $\frac{-39}{1}$, $\frac{78}{-2}$ **, 0.00**

b. A fraction in lowest terms (when such format is requested)

Examples of acceptable answers:

$\frac{2}{7}$, $\frac{-3}{7}$, $\frac{3}{-7}$, $-\frac{5}{6}$, $+\frac{5}{6}$

Examples of unacceptable answers:

$\frac{4}{14}$, $\frac{3.3}{66}$, $\frac{\frac{2}{3}}{6}$, $1\frac{7}{10}$, $\frac{2.0}{7}$

c. A decimal number correct to specified number of decimal places
(such an answer is accepted only when such format is requested)
Examples of acceptable answers (correct to one decimal place):
11.3, 2.0, -4.7, .3
Examples of acceptable answers (correct to three decimal places):
11.312, 2.000, -4.717, .010
Examples of unacceptable answers (correct to one decimal place):
11.31, 2., -4.71, $\frac{3}{6}$**, 3+0.1**
Examples of unacceptable answers (correct to three decimal places):
11.3112, 2.20, -4.7100, $\frac{3}{8}$

Other formats of answers if such is requested

Some questions include units and for these questions the answer must be provided in specified units
Examples: year, day, hour, minute, second, km, metre, cm, mm, m^2, cm^2, litre, m^3, cm^3, kg, g, $\frac{km}{s}$, $\frac{m}{s}$, \$, cent, etc…
The answer must be provided in the specified units.
The unit's symbol will be provided.
Students are strongly asked not to write any unit next to their answer.

If more than a single answer is provided to a question, the markers will mark it as wrong even if one of the answers provided by the student is correct (0 marks).

Basic geometric shapes and formulas

Circle (radius r)
Circumference: $2\pi r$
Area: πr^2

Triangle (sides a, b, c, angles A, B, C)
Perimeter: $a + b + c$
Sum of angles: $A + B + C = 180°$
Note that for every tringle: $a + b > c$

Right triangle ($C = 90°$, $a^2 + b^2 = c^2$)
Area: $\frac{ab}{2}$

Isosceles triangle ($a = b$, $A = B$)

Equilateral triangle
($a = b = c$, $A = B = C = 60°$)
Note that equilateral triangle is also isosceles triangle.

Problem 1

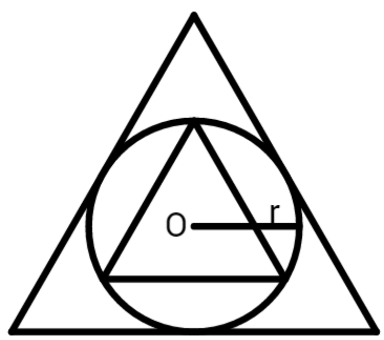
An equilateral triangle is circumscribed inside a circle with radius $r = 20$. What is the area of the triangle?

Approximate $3\sqrt{3} = 5.196$ and round the answer to the nearest integer

Problem 2

A circle with radius $r = 20$ is circumscribed inside an equilateral triangle. What is the area of the triangle?

Round the answer to the nearest integer.



Problem 3

A circle with radius $r = 20$ is circumscribed inside an isosceles right triangle. What is the area of the triangle?

Approximate $2\sqrt{2} = 2.828$ and round the answer to the nearest integer.

Answers

Problem 1

Side of the triangle

$$2\sqrt{r^2 - \frac{r^2}{4}} = 2\,\frac{r\sqrt{3}}{2} = r\sqrt{3}$$

Height of the triangle $r + \frac{r}{2} = \frac{3r}{2}$

Area

$$r\sqrt{3} \times \frac{3r}{4} = r^2\sqrt{3}\,\frac{3}{4} = \frac{20^2}{4} \times 5.196 = 520$$

Problem 2

Side of the triangle

$$2\sqrt{(2r)^2 - r^2} = 2r\sqrt{3}$$

Height of the triangle $2r\sqrt{3} \times \frac{\sqrt{3}}{2} = 3r$

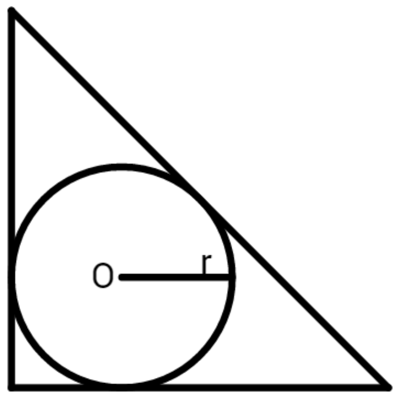
Area

$$2r\sqrt{3} \times \frac{3}{2}r = r^2 3\sqrt{3} =$$

$$400 \times 5.196 = 2078.4,$$

so the area is 2078

Problem 3



Distance to centre of circle from the 90° corner of the triangle $r\sqrt{2}$

Thus, side of the triangle $(r + r\sqrt{2})\sqrt{2}$

Area $\frac{1}{2}\left(r\sqrt{2}\left(1+\sqrt{2}\right)\right)^2 =$

$r^2\left(1 + 2\sqrt{2} + 2\right) = r^2(3 + 2\sqrt{2})$

So, for $r = 20$, area is

$400 \times (3 + 2.828) = 400 \times 5.828$

$400 \times 5.828 = 2331.2,$

so answer is 2331

Obtuse triangle ($C > 90°$)

Square
(side a, angle $A = 90°$)
Perimeter: $4a$
Area: a^2
Number of diagonals: 2
Length of diagonals: $\sqrt{2}a$

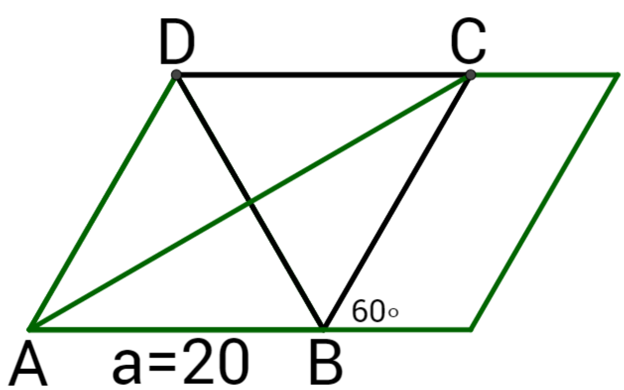
Rectangle
(sides a, b, all angles $A = 90°$)
Perimeter: $2(a + b)$
Area: ab
Number of diagonals: 2
Length of diagonals: $\sqrt{a^2 + b^2}$

Parallelogram
(sides $a = c, b = d$, $a \parallel c, b \parallel d$,
angles A, B, $A + B = 180°$)
Number of diagonals: 2

Trapezoid
(sides a, b, c, d, $a \parallel c$,
$A + B = 180°, C + D = 180°$)

Rhombus
(side a)
Diagonals: 2, perpendicular, $d_1 \parallel d_2$
Area: $\frac{d_1 d_2}{2}$

Problem 4
Two parallel sides of a rhombus with side $a = 20$ and angle $A = 60°$ are extended by 50% to make a parallelogram. What is the area of the parallelogram? Approximate $\sqrt{3} = 1.732$ and round your answer to the nearest integer.



Answer

Problem 4

Diagonals of the rhombus:

$$d_1 = a = 20, d_2 = 2\frac{a\sqrt{3}}{2} = a\sqrt{3}$$

Area of rhombus

$$\frac{1}{2}(d_1 \times d_2) = a^2\frac{\sqrt{3}}{2}$$

Area of parallelogram

$$\frac{3}{2}a^2\frac{\sqrt{3}}{2} = \frac{3}{4} \times 400 \times 1.732 \approx 520$$

Convex polygon with n sides

(all angles $A_i, i = 1, 2, 3, \cdots, n$, satisfy $0 < A_i < 180°$)

Number of diagonals from each corner:

$n - 3$

Total number of diagonals: $\frac{n(n-3)}{2}$

Sum of all angles: $(n - 2)180°$

Regular polygon with n sides

(all angles $A_i, i = 1, 2, 3, \cdots, n$, have the same value $A_i = \frac{180(n-2)}{n}°$)

Regular Pentagon
($n = 5, A_i = 108°$)
Regular Hexagon
($n = 6, A_i = 120°$)

Problem 5
The number of diagonals of a regular polygon is 209.
a) How many sides does the polygon have?
b) What is the value, in degrees, of the angle between each of the sides of the polygon?
Express the answer as a fraction in lowest terms.

Answer

Problem 5

a) $\frac{1}{2}n(n-3) = 209.$

Since $209 = 11 \times 19$, $n = 22$.

b) Total sum of all angles is

$180 \times 20 = 3600°.$

Thus, $A_i = \frac{3600}{22} = \frac{1800}{11}°$

Sphere

(3-D symmetrical body, i.e. a ball with radius r)

Rectangular box, or a prism

(all edges are either perpendicular or parallel to all other edges)

Number of edges: 12

Number of faces: 6

Three mutually perpendicular edges a, b, c

Sum of all edges: $4(a + b + c)$

Surface area: $2(ab + ac + bc)$

Volume: abc

Pyramid
(a body with base in the shape of regular polygon and a top vertex directly above the centre of the polygon)

Cone
(a body with base in the shape of circle a top vertex directly above the centre of the circle)

Problem 6
a, b, c **are the mutually vertical edges of a rectangular box. Let** $a + b + c = 40$**, and given that** a, b, c **are different primes,**
a) What is the volume of the box?
b) What is the surface area of the box?

Problem 7
A pyramid has 53 **faces.**
a) How many edges does it have?
b) How many diagonals does the base have?

Answers

Problem 6

a) It is clear that $a = 2$.
Thus, $b + c = 38$. Consider all primes less than 20. For $b = 3, 5, 11, 13, 17$, c is not prime. For $b = 19$, $c = 19$.
Thus, $b = 7$, $c = 31$.
$$a \times b \times c = 2 \times 7 \times 31 = 434.$$
b) Surface area: $2(ab + ac + bc)$.
So, $2(14 + 62 + 217) = 2 \times 293 = 586$

Problem 7

a) The base is a polygon with 52 sides. So the total number of edges is 104.
b) The number of diagonals of the base:
$$26 \times 49 = 1274.$$

Arithmetic sequence
$\{a_1, a_2, a_3, \cdots, a_n, \cdots\}$ is defined in the following way:
$a_1 = k$, for $n > 1$, $a_n - a_{n-1} = d$,
k and d are constants.
Geometric sequence $\{a_1, a_2, a_3, \cdots, a_n, \cdots\}$ is defined in the following way:
$a_1 = k$, for $n > 1$, $a_n = a_{n-1} \times p$,
k and p are constants.

Problem 8
The terms $a_1 = 4$, $a_9 = 1024$ are the first and ninth terms of an arithmetic sequence with constant d, and are also the first and ninth terms of a geometric sequence with constant $p < 0$.
a) Find d. Express d as a decimal correct to 1 decimal place.
b) Find p.
c) Find the sum of the first 5 terms of the arithmetic sequence.
d) Find the sum of the first 5 terms of the geometric sequence.

Answer
Problem 8

a) $a_9 - a_1 = 8d$. $d = \frac{1024-4}{8} = 127.5.$

b) $p^8 = \frac{a_9}{a_1} = \frac{1024}{4} = 256$. **So,** $p = -2.$

c) Sum of the first 5 terms of the arithmetic sequence is given by:

$$(a_1 + a_5)\frac{5}{2} = (4 + 4 + 4 \times 127.5)\frac{5}{2} = 4$$

Sum is $(1 + 1 + 127.5) \times 10 = 1295$

d) grouping: $(4 - 8) + (16 - 32) + 64,$ **the sum is** $a_1 - 4 - 16 + 64 = 44.$

In general, if S_n **is the sum of the first** n **terms of a geometric sequence then** S_n **can be found in the following way:**

$$(q-1)S_n = (q-1)(a_1 + \cdots + a_n) =$$
$$a_2 - a_1 + \cdots + a_{n+1} - a_n =$$
$$a_{n+1} - a_1 = a_1(q^n - 1)$$

So, $S_n = \frac{a_1(q^n-1)}{q-1}$

repeated decimals

Any fraction can be written as repeated decimal.

Examples:

$\frac{1}{3} = 0.333\cdots$

$\frac{4}{11} = 0.363636\cdots$

$\frac{11}{14} = 0.785714285714285714 2\cdots$

Problem 9

x is a fraction in lowest terms that satisfies: $x = 0.3515151\cdots$

Find x.

Answer

Problem 9

$(1000-10)x =$

$351.515151\cdots - 3.515151\cdots =$

$351 - 3 = 348$

$x = \frac{348}{990} = \frac{174}{495} = \frac{58}{165}$

Note that: $58 = 2 \times 29$ **,** $165 = 5 \times 33$

Sometimes this technique to find a fraction is hard when you have to factor large numbers.

For $\frac{11}{14} = 0.7857142857142857142\cdots$

With this method:

$(10,000,000-10)x =$

$7,857,142.857142\cdots - 7.857142\cdots =$

$7,857,135$**. So,**

$$x = \frac{7,857,135}{9,999,990} = \frac{873015}{1,111,110} =$$

$$\frac{174603}{222,222} = \frac{58201}{74074} = \frac{4477}{5698} = \frac{407}{518} = \frac{11}{14}$$