Introduction to Problem Solving

Melania Alvarez
University of British Columbia

Four Principles

How to Solve It suggests the following steps when solving a Mathematical Problem:

- Understand the problem
- Make a plan
- Carry out the plan
- Look back on your work. How could it be better?

Second principle: Devise a plan

- 1. Use Diagrams / Models
  2. Act it Out
  3. Use Before & After
  4. Use Systematic Listing
  5. Look for Patterns
  6. Work Backwards
  7. Use Guess & Check
  8. Simplify the Problem
  9. Make Supposition
  10. Solve Part of the Problem
  11. Paraphrase the Problem

Danny raises some chickens and rabbits in this little farm. These animals have 15 heads and 40 feet altogether. How many chickens and rabbits does he raise?

Step One: Understand

- How many chickens and rabbits are there altogether?
- How many feet are there altogether?
- How many feet does each chicken have?
- How many feet does each rabbit have?

Problem Solving

Problem —> Diagram —> Calculation

- ***Mathematical concepts or problems sometimes require an illustrative calculation.***
- ***Mathematical Concepts can occur in a variety of situations, giving the opportunity to use what is known.***

Kathy bought a blouse for three times as much as her scarf cost. The scarf cost half as much as her hat. Her hat cost $10.00. How much did her blouse cost?

Use of these models to solve problems

2/5 of a number equals 12. What is the number?

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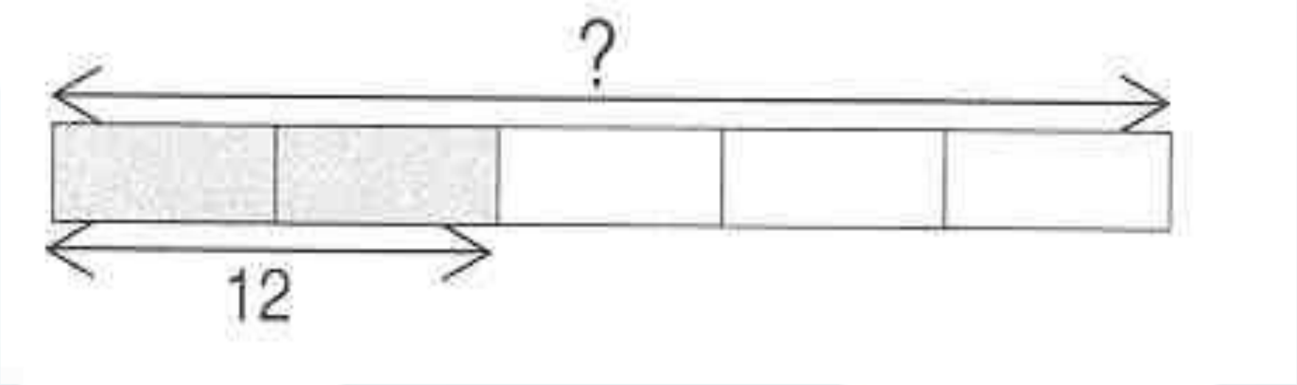

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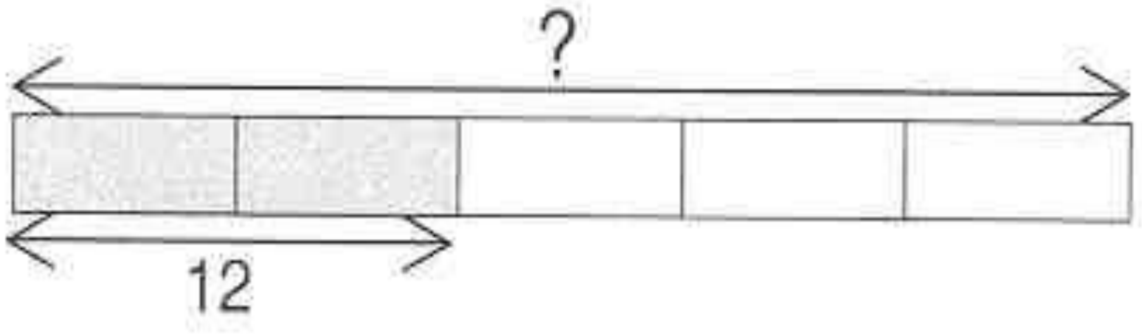

2 units ——> 12

1 unit ——> $12 \div 2 = 6$

5 units ——> $5 \times 6 = 30$

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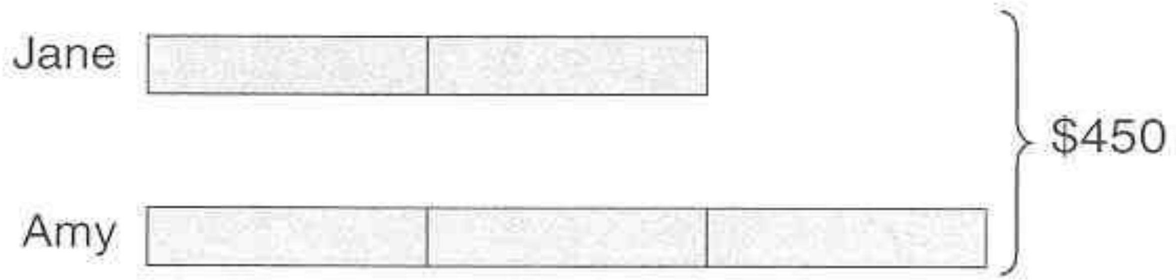

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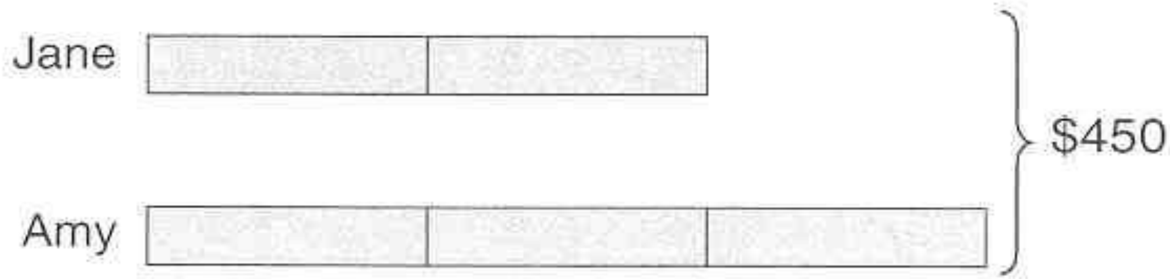

5 units ——> $450

1 unit ——> $450 ÷ 5 = $90

2 units ——> 2 x $90 = $180

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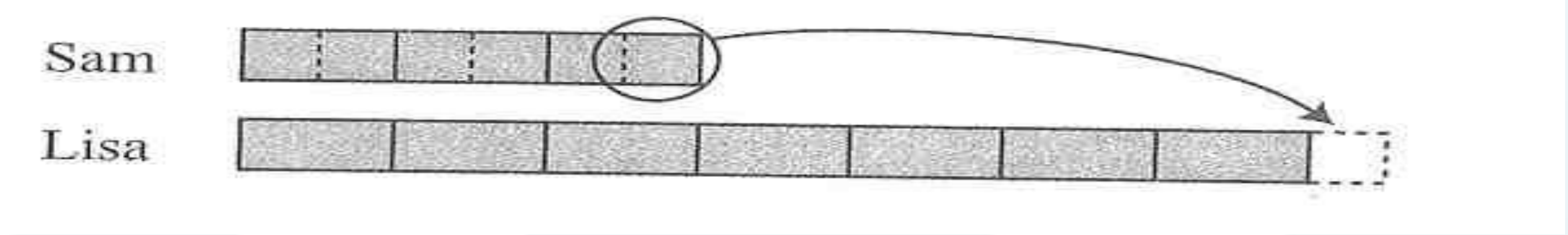

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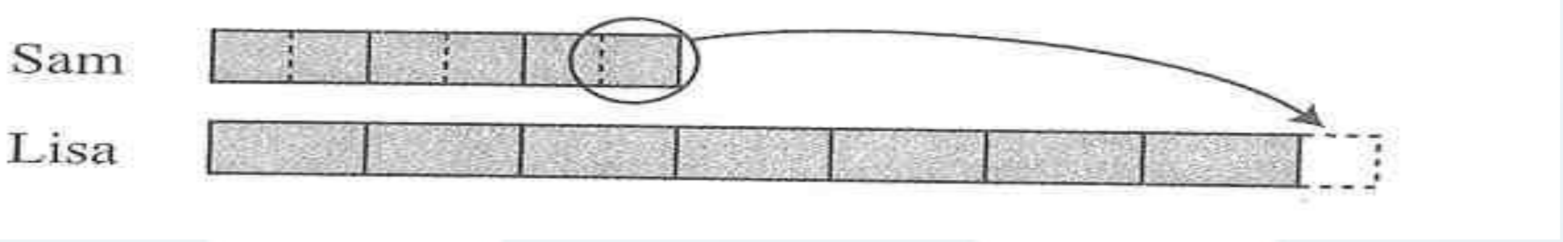
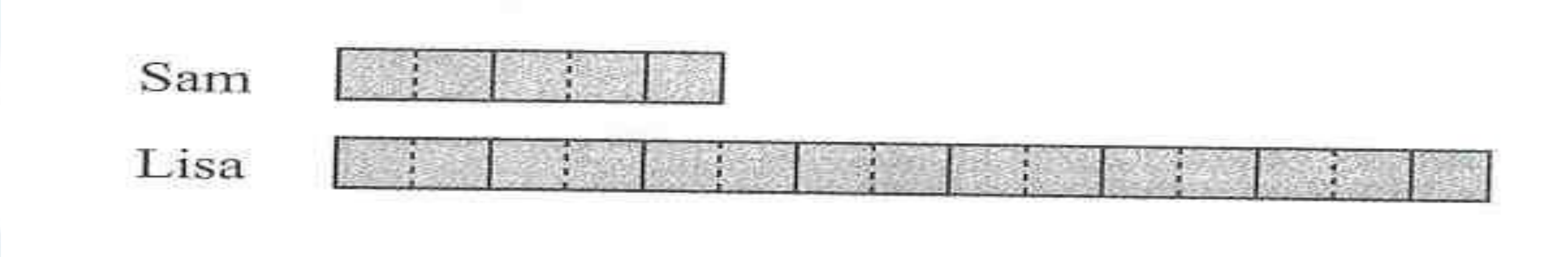
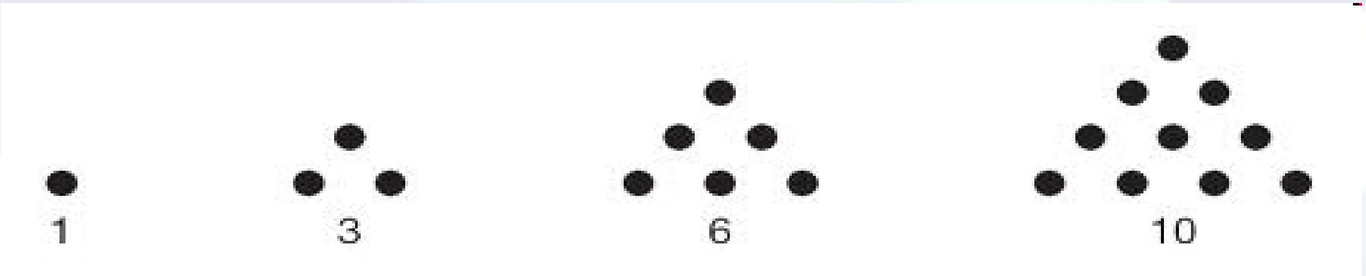


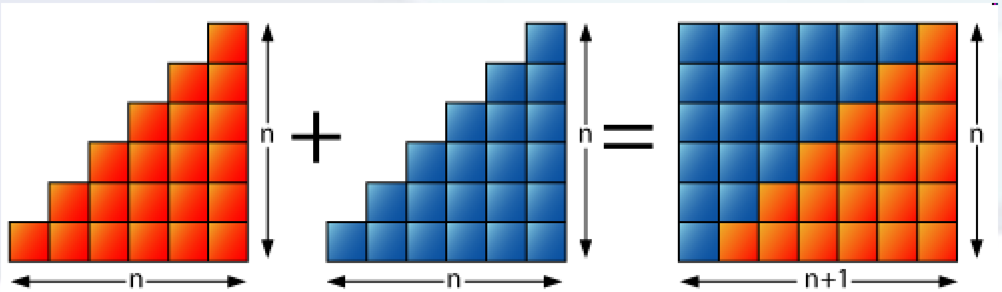
After





The first four triangular numbers are 1, 3, 6 and 10.

What is the 10th triangular number?



- ***Let's find the value of :***

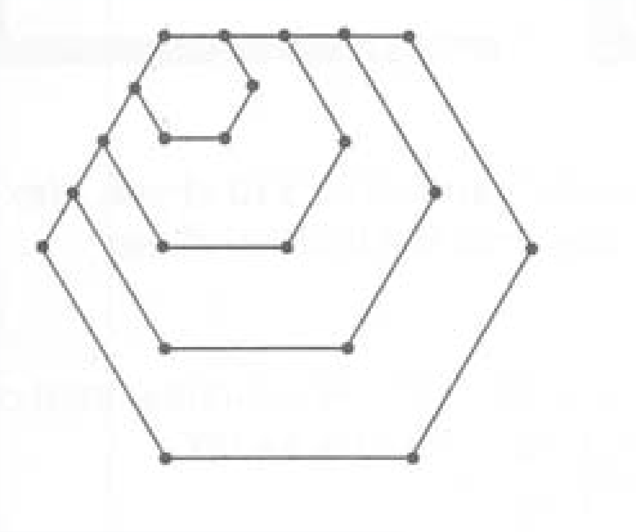
1 - 2 + 3 - 4 + 5 - 6 +…. 99 – 100

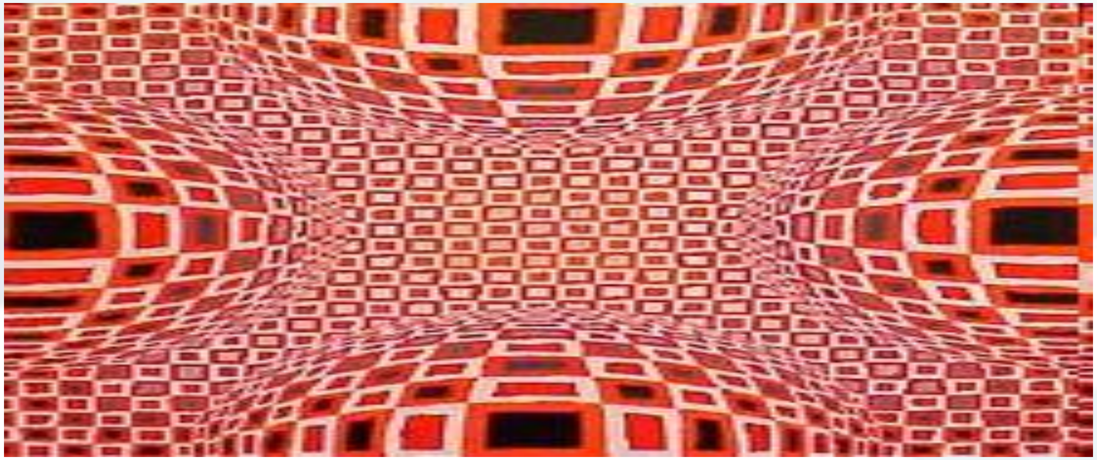
- ***Let's find the value of :***

1 + 2 + 3 + 4 + 5 + 6 +…. 99 + 100

Find the sum of 11 + 13 + 15 + ……..+ 99

Robert is bored and started drawing one hexagon, then kept drawing larger and larger hexagons. How many dots he would have altogether after the 8th hexagon?





A piece of wire 52cm long is cut into two parts.

Each part is then bent to form a square. The total area of the two squares is 97 cm2.

How much longer is a side of the larger square than a side of the smaller square ? (Consider only integers for the lengths of the sides.)

Solve for the variable.

$2(x + 5) - 7 = 3(x - 2)$.

$$2(x+5)-7=3(x-2)$$

$$2x+10-7=3x-6$$

$$2x+3=3x-6$$

$$2x+3-3x=3x-6-3x$$

$$-x+3=-6$$

$$-x+3-3=-6-3$$

$$-x=-9$$

$$\frac{-x}{-1}=\frac{-9}{-1}$$

$$x=9$$

Question: Solve the system of equations:

$3x+4y=10$

$2x-y=3$

Solve the second equation for y

$2x-y=3$

$y=2x-3$

Substitute into the first equation:

$3x+4(2x-3)=10$

$3x+8x-12=10$

$11x=22$

$x=2$

Substitute $x=2$ into $y=2x-3$

$y=2(2)-3=4-3=1$.

Answer: $x=2$ $y=1$

A man is 4 times as old as his son. In 20 years, the man will be twice as old as his son. How old are the man and his son currently?

Let the present age of the son be x, and the present age of the man be 4x.

In 20 years, the son's age will be x + 20, and the man's age will be 4x + 20.

According to the problem, the man's age will be twice the son's age in 20 years:

$4x+20=2(x+20)$

Simplifying:

$4x+20=2x+40$

$4x-2x=40-20$

$2x=20$

$x=10x$

Therefore, the son's present age is 10, and the man's present age is 40.

Answer: The son is **10 years old**, and the man is **40 years old**.

Question: A machine can complete a task in 10 hours, while another machine can complete the same task in 15 hours. How long will it take them to complete the task together?

Work done per hour by the first machine = 1/10

Work done per hour by the second machine = 1/15

Together, the rate of work = 1/10 + 1/15 = 5/30 + 2/30 = 7/30.

Thus, the time taken to complete the task together is the reciprocal of 7/30 = 30/7 hours, which is approximately 4.29

A tank can be filled by Pipe A in 10 hours and by Pipe B in 15 hours. Pipe C, which drains the tank, can empty the tank in 12 hours. If all three pipes are opened at the same time, how long will it take to fill the tank?

The rate of work for each pipe is:

- Pipe A: 1/10 tank per hour
- Pipe B: 1/15 tank per hour
- Pipe C: −1/12 tank per hour (since it drains the tank)
- The combined rate is:
- 1/10 + 1/15 -1/12

LCM of 10, 15, 12 is 60

So

1/10 + 1/15 -1/12 =

6/60 + 4/60 -5/60 = 5/60

Therefore , the time to fill the tank is

60/5 hours = 12 hours

Percent means per 100, or divided by 100. Dividing by 100 moves the decimal point two places to the left.

$$24\% = \frac{24}{100} = .24$$

To convert a fraction or decimal to a percentage, multiply by 100:

Multiply the fraction by 100 to give the result as a percentage value.

$$\frac{1}{5} \times 100 = 5\overline{)100} = 20$$

To convert a percent to a fraction, divide by 100 and reduce the fraction (if possible):

Divide the percentage value by 100 and simplify the fraction if necessary.

$$60\% = \frac{60}{100} = \frac{3}{5}$$

12 people out of a total of 25 were female. What percentage were female?

Multiply by 100. Dividing the top and bottom by 25 (cancelling) leaves 12 x 4.

$$\frac{12}{\not{25}} \times \overset{4}{\not{100}} = 48\%$$

The price of a $1.50 candy bar is increased by 20%. What was the new price?

Multiply the price by 20% (20/100). Add the result to the original price. ($1.50 + .30 = $1.80)

$$\$1.50 \times \frac{20}{100} = \$0.30$$

$$\$1.50 + \$0.30 = \$1.80$$

The tax on an item is $6.00. The tax rate is 15%. What is the price without tax?

The price, p, times 15% (15/100) equals 6. Solve the equation by multiplying both sides by 100 and then dividing both sides by 15. The price without tax (P) is 40.

$$P \times \frac{15}{100} = 6$$

$$P \times \frac{15}{\cancel{100}} \times \cancel{100} = 6 \times 100$$

$$P \times \frac{\cancel{15}}{\cancel{15}} = \frac{600}{15} = 40$$

A brand new movie just came to Golden Theater.

The theater has a goal of selling 2000 tickets for this movie on the 1st week.

The first 2 days it sold 30% of its goal tickets. What is the average number of tickets the theater needs to sell in the next 5 days in order to meet their goal?

The next 5 days sales needs to be 2000 x (100% -30%) = 1400 tickets;

Average daily sales = 1400 / 5 = 280 tickets

October 1st of 2025 will be on Wednesday. What day of the week is Christmas day, December 25th, 2025?

Solution:

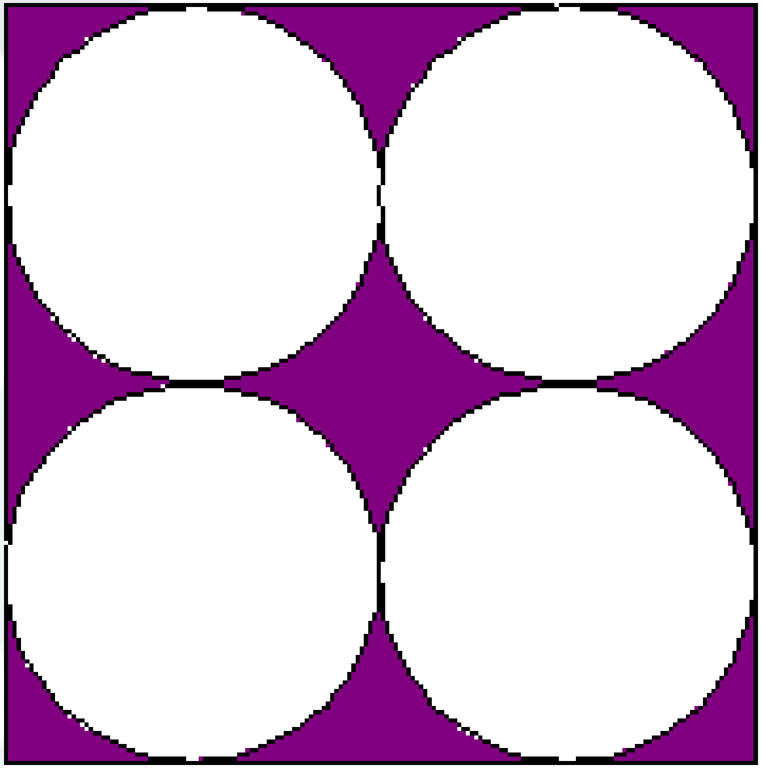
There are 31 + 30 + 25 -1 = 85 days from October 1st to December 25th. There are 7 days in a week. 85 ÷7 = 12 remainder 1; Therefore December 25th is on a Thursday.

If a car dealership gives a 5% discount on a car, the dealership will make a $5250 profit on the car.

If, instead it will give a 25% discount, the dealership will lose $1750.

How much did the dealership pay for the car (in dollars)?

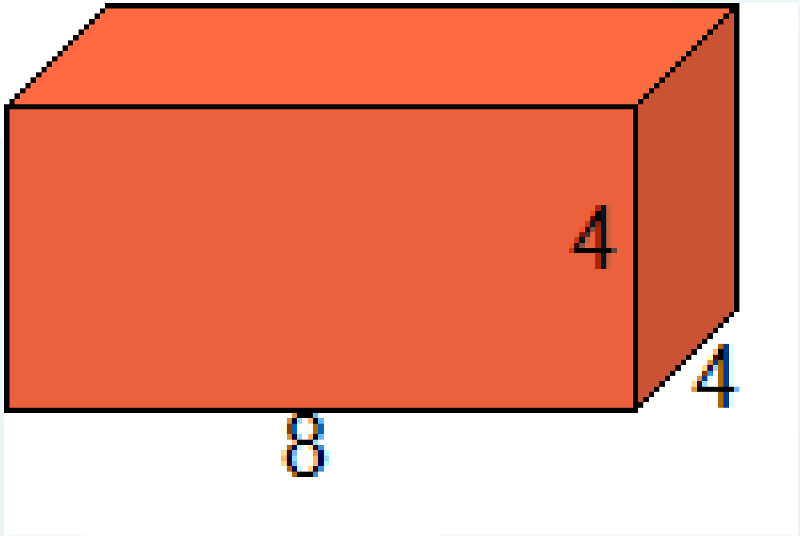
The diameter of the circles is 4 centimeters.
Calculate the area in purple.



The area is

8 x 8 - 4 x (3.14 x 2 x 2) cm square

A rectangular prism is 8 centimeters long, 4 centimeters wide and 4 centimeters tall.



How many 2 x 2 x 2 cm cubes can the prism be cut into?

What is the total surface area of all the 2 x 2 x 2 cm cubes?

What is the total volume of all the 2 x 2 x 2 cm cubes?

A train is 300 meters long. A tunnel is 2900 meters long.

- The train travels at 600 meters per minute.
- How long is the time between the moment the front of the train enters the tunnel and the moment the end of the train exits the tunnel?

The length the train needs to travel is

2900 + 300 =3200 meters,

Therefore, it takes

3200 ÷ 600 = 8 minutes for the train to pass the tunnel.

Find the number of cubes in each solid.

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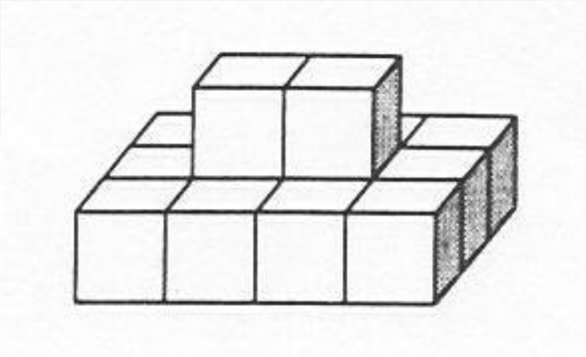
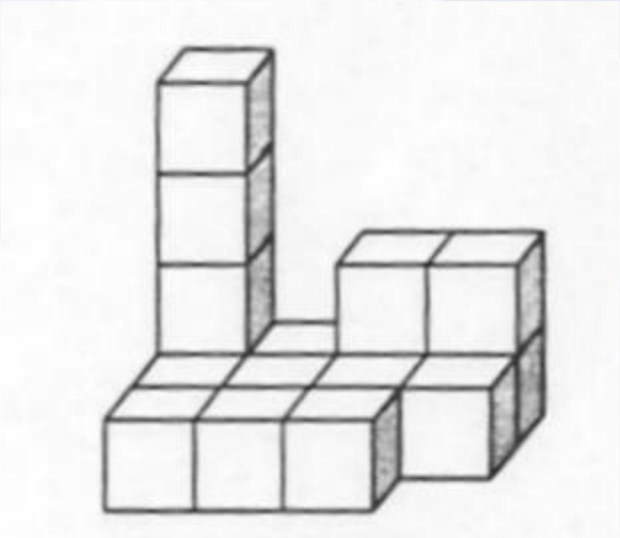
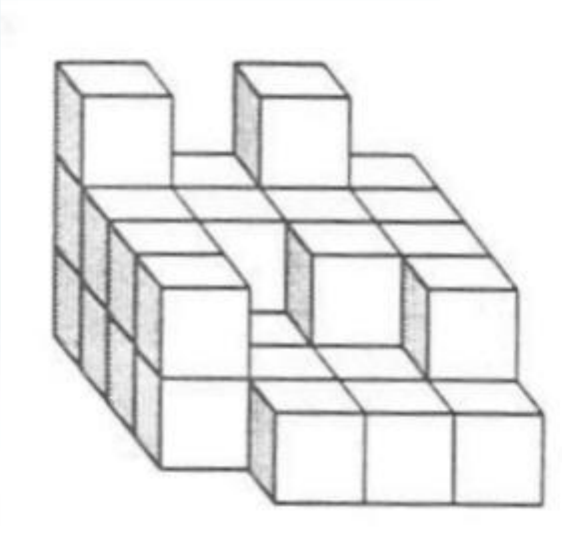






What is an arithmetic sequence?

An **arithmetic sequence** is an ordered set of numbers that have a **common difference** between each consecutive term.

For example in the arithmetic sequence 3, 9, 15, 21, 27, the common difference is 6.

An arithmetic sequence can be known as an arithmetic progression. The difference between consecutive terms is an arithmetic sequence is always the same.

If we **add** or **subtract** by the **same number** each time to make the sequence, it is an **arithmetic sequence**.

The **term-to-term rule** tells us how we get from one term to the next.

Here are some examples of arithmetic sequences:

Here are some examples of arithmetic sequences:

First Term	Term-to-Term Rule	First 5 Terms
3	Add 6	3, 9, 15, 21, 27, …
8	Subtract 2	8, 6, 4, 2, 0, …
12	Add 7	12, 19, 26, 33, 40, …
-4	Subtract 5	-4, -9, -14, -19, -24, …
½	Add ½	½, 1, 1½, 2, 2½, …

Arithmetic Sequence formula:

$a_n = a_1 + (n-1)d$

$$3,\ 9,\ 15,\ 21,\ 27,\ \ldots$$

+6 +6 +6 +6

$$a_1 = 3$$
$$a_2 = 3 + 6$$
$$a_3 = 3 + (\underline{2} \times 6)$$
$$a_4 = 3 + (\underline{3} \times 6)$$
$$a_5 = 3 + (\underline{4} \times 6)$$
$$\vdots$$
$$a_n = a_1 + (\underline{n - 1})d$$

Examples:

Calculate the next three terms for the sequence 4, 7, 10, 13, 16, …

Calculate the next three terms for the sequence -3, -9, -15, -21, -27, …

Calculate the next 3 terms of the sequence 5, 3, 1, −1, −3, …5

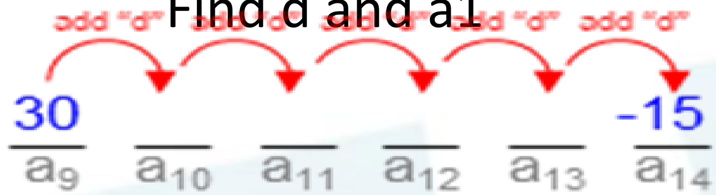
By finding the common difference, state the next 3 terms of the sequence −37,−31,−25, −19,−13,…

More examples:

Calculate the sum of the 1^{st} ,10^{th}, 100^{th} ,and 1000^{th} term of the sequence $4n-25$

An arithmetic sequence means you are adding some number to each term to get to the next term.

Find d and a1



Can you see that to go from a9 to a14, we need to add "d" five times therefore:

$30 + 5d = -15$

$5d = -15 - 30$

$5d = -45$

$d = -45/5$

$d = -9$

Using the equation for an arithmetic sequence: $a_n = a_1 + d(n - 1)$ we can use the fact that $a_9 = 30$ and plug the values of $n = 9$ and $a_9 = 30$ into the equation:

$30 = a_1 + (-9)(9 - 1)$

$30 = a_1 + (-9)(8)$

$30 = a_1 + -72$

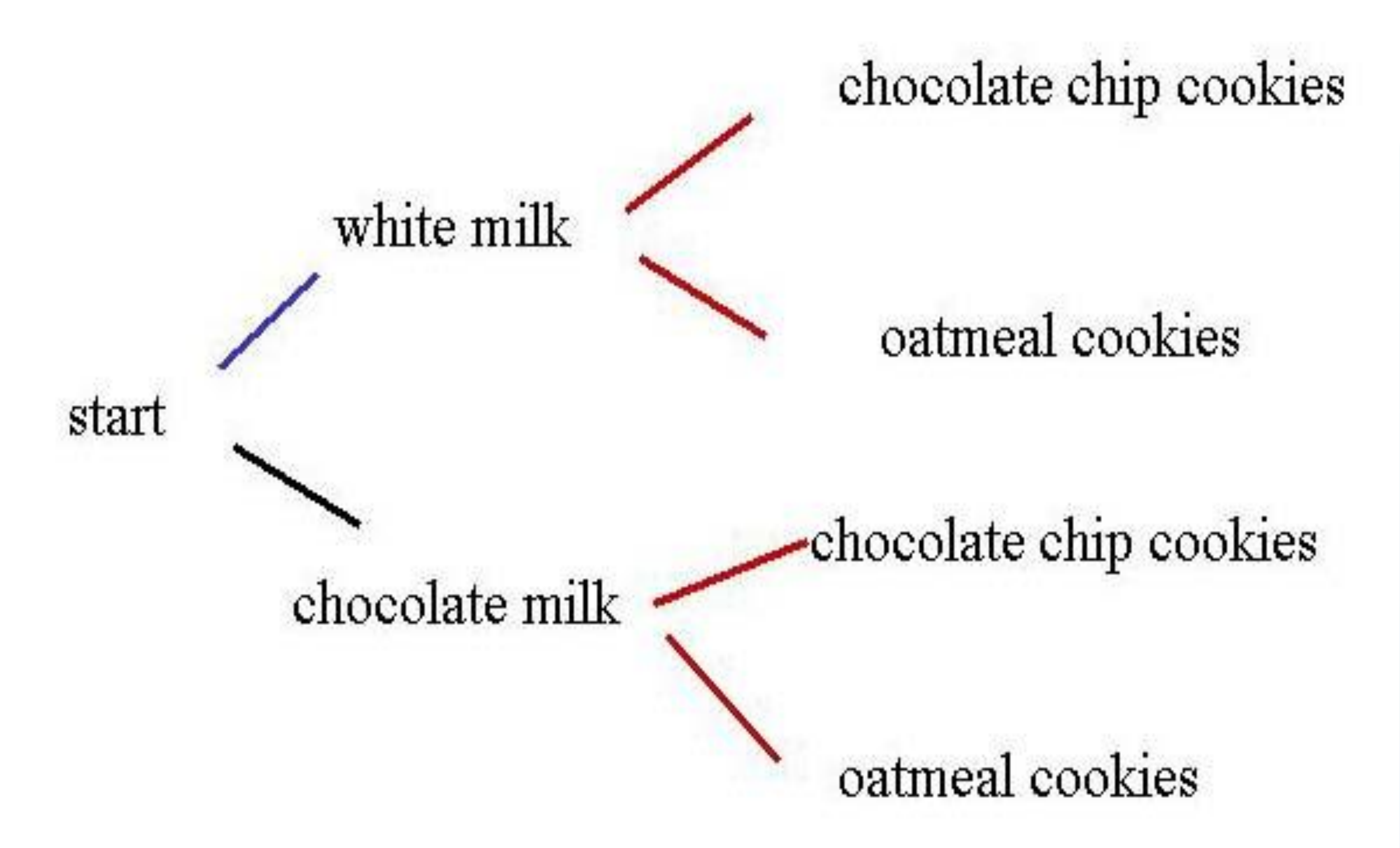
$30 + 72 = a_1$

$a_1 = 102$

Tree Diagram

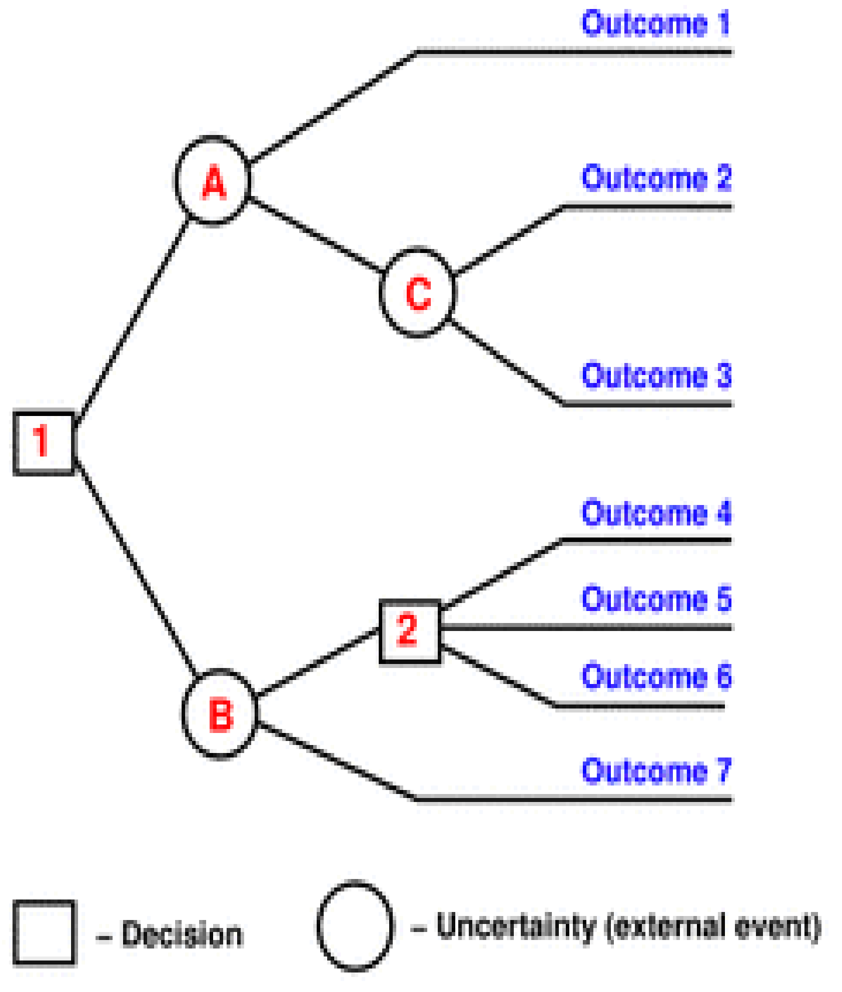
A tree diagram is simply a way of representing a sequence of events. Tree diagrams are particularly useful in probability since they record all possible outcomes in a clear and uncomplicated manner.

Tree Diagram?



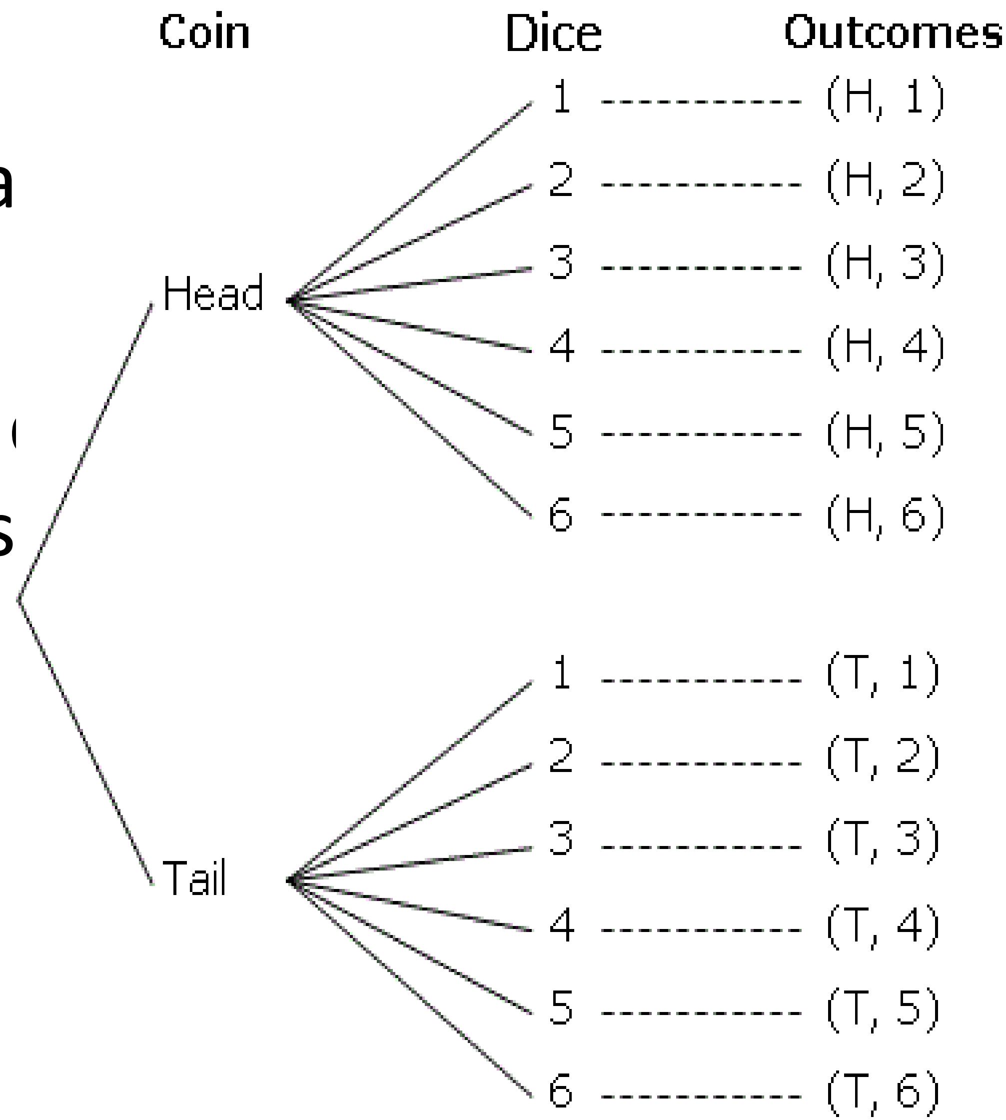
Decision Tree

- Decision Trees help you choose between multiple outcomes/courses you might take. They are very visual and help the user understand the risks and rewards associated with each choice.



Probability problems

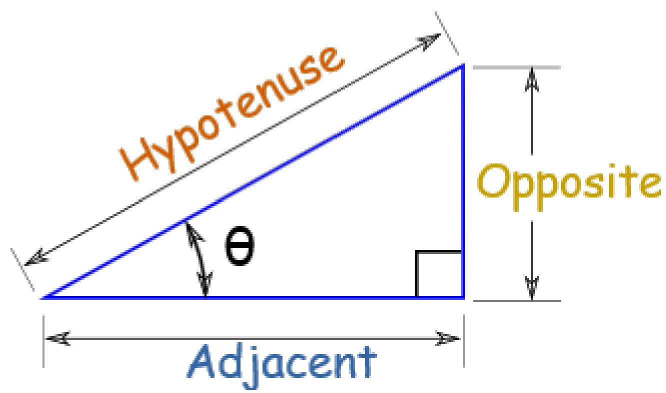
- A coin and a dice a
- We can use a tree
possible outcomes



Trigonometry

degrees	radians	sin θ	cos θ	tan θ	csc θ	sec θ	cot θ
0°	0	0	1	0	–	1	–
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
45°	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$
90°	$\frac{\pi}{2}$	1	0	–	1	–	0

- Sine: sin(θ) = Opposite / Hypotenuse
- Cosine: cos(θ) = Adjacent / Hypotenuse
- Tangent: tan(θ) = Opposite / Adjacent



If $\sin\theta = 3/5$, find $\cos\theta$