Elmacon

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Divisibility Lemma:

- Suppose A is a number divisible by k. $\Longleftrightarrow$ The B is divisible by k A+B is divisible by k.

- Suppose A is divisible by k. $\Longrightarrow$ If B is divisible by k

  A + B is divisible by k

- Suppose A is divisible by k. If A +B is divisible by k $\Longrightarrow$ B is divisible by k

The Division is very powerful.

<u>Ex</u> Is 384 divisible by 6?

$$\left.\begin{array}{r}360 \text{ is} \\ 24 \text{ is}\end{array}\right\} \underset{\text{Div. Lemma}}{\Longrightarrow} 384 \text{ is!}$$

<u>Ex</u> Is 454 divisible by 9?

$$\left.\begin{array}{r}450 \text{ is} \\ 4 \text{ isn't}\end{array}\right\} \underset{\text{Div. Lemma}}{\Longrightarrow} 454 \text{ isn't.}$$

Use the lemma:

- Is 4284 divisible by 6?

Is 168 divisible by 8?

3648 and 4260 are divisible by 6, is 3648 + 4260 divisible by 6?

Divisibility Rules	
A number is divisible by	
2	If last digit is 0, 2, 4, 6, or 8
3	If the sum of the digits is divisible by 3
4	If the last two digits is divisible by 4
5	If the last digit is 0 or 5
6	If the number is divisible by 2 and 3
7	cross off last digit, double it and subtract. Repeat if you want. If new number is divisible by 7, the original number is divisible by 7
8	If last 3 digits is divisible by 8
9	If the sum of the digits is divisible by 9
10	If the last digit is 0
11	Subtract the last digit from the number formed by the remaining digits. If new number is divisible by 11, the original number is divisible by 11
12	If the number is divisible by 3 and 4

Divisibility by 3

A whole number is said to be divisible by 3 if the sum of all digits of that whole number is a multiple of 3 or exactly divisible by 3.

- In 1377, the sum of all the digits = 1+3+7+7 = 18.
  - Since 18 is divisible by 3, it means 1377 is also divisible by 3.
  - Here, 1377 ÷ 3 = 459 is the quotient and the remainder is 0.

- Is 2130 divisible by 3?
  - In 2130, the sum of all the digits = 2+1+3+0 = 6.
  - Since 6 is divisible by 3, it means 2130 is also divisible by 3.
  - Here, 2130 ÷ 3 = 710 is the quotient and the remainder is 0.

- Is 3194 divisible by 3?
  - In 3194, the sum of all the digits = 3+1+9+4 = 17.
  - Since 17 is not divisible by 3, it means 3194 is not exactly divisible by 3 ⇒ 3194 ÷ 3 = 1064 is the quotient and the remainder is 2.

What is the Divisibility Rule of 4?

- According to the divisibility rule of 4, a whole number is said to be divided by 4 if it has fulfilled one of the two conditions:
- The last two digits in a 3-digit whole number are zeros this means the number has zeros at tens place and ones place.
- The last two digits of a whole number form a number that is exactly divisible by 4.

- Is 1124 divisible by 4?

  - In 1124, the last two digits at tens place and ones place form a number 24 which is divisible by 4 (24 ÷ 4 = 6)

  - Thus, 1124 is divisible by 4. 1124 ÷ 4 = 281

- a.) Is 117 divisible by 4?
  - In 1171 the last two digits at tens place and ones place form a number 71 which is not exactly divisible by 4 (71÷ 4 = 17 is quotient and remainder is 3)
    Thus, 1171 is not divisible by 4.

- b) Is 1300 divisible by 4?
  - In 1300 the last two digits at tens place and ones place are two zeros. That means 1300 is exactly divisible by 4.
    1300 ÷ 4 = 325
    Thus, 1300 is divisible by 4.

Divisibility Rule of 7 Example

Is 2455 divisible by 7?

245(5) → Multiply the last digit by 2

\- 10 → Subtract the product (10) from 245

....

235 → Is 235 a multiple of 7? We are unsure about it. Let us repeat the process further.

23(5) → Multiply the last digit by 2.

\- 10 → Subtract the product (10) from 23

....

13 → Is 13 a multiple of 7?

No, so 2455 is not divisible by 7.

Divisibility by 8

Divisibility Rule of 8

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a) The last 3 digits should be divisible by 8

83416

⟹ last 3 digits : 416 ✓

$416 \div 8 = 52$

∴ 83416 is divisible by 8

OR

b) The last 3 digits should be 000

52000

⟹ last 3 digits : 000 ✓

∴ 52000 is divisible by 8

Divisibility Rule of 9

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THE MATH EXPERT

7209	2807
Sum of digits = 7 + 2 + 0 + 9	Sum of digits = 2 + 8 + 0 + 7
= 18 ✔	= 17 ✘
18 is divisible by 9	17 is not divisible by 9
⇒ 7209 is divisible by 9	⇒ 2807 is not divisible by 9

Divisibility Rule of 11

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THE MATH EXPERT

Step 1 : Start from the leftmost or the rightmost digit.

Step 2 : Find the sum of all the digits at the odd positions.

Step 3 : Find the sum of all the digits at the even positions.

Step 4 : Find the difference between the sum obtained in step 2 and step 3.

Step 5 : If the difference is 0 or a number that 11 can divide completely without leaving a remainder, then the number is divisible by 11.

- In the divisibility rule of 11, we check to see if the difference between the sum of the digits at the odd places and the sum of the digits at even places is equal to 0 or a number that is divisible by 11
- **Number = 764852**
- Let us check if divisibility by 11 is true.

Sum of the digits at odd places (from the left) = 7 + 4 + 5 = 16
Sum of the digits at even places = 6 + 8 + 2 = 16
Difference between the sum of the digits at odd and even places = 16 - 16, which is 0.
Therefore, 764852 is divisible by 11.

- Which numbers below that divide the number 117,645.
- 3 4 5 8 9 11
-
-
- Which numbers below are factors of the number 3828.
- 2 3 4 5 8 9 10 11
-
-

- Suppose that a 6-digit number is divisible by 11. If the digits are rearranged to form a new number, is this new number also divisible by 11? If yes, give an explanation. If no, give a counterexample.

- The letters a and b are digits (i.e., 0, 1,…,9). Find an a and b so that 41a2330b0 is divisible by 8, but not divisible by 3.
-
- Find three pairs of digits a,b so that the number 5a31411b2 is divisible by 6, but is not divisible by 9. Explain your reasoning.
-
-
-

- Suppose that a 6-digit number is divisible by 11. If the digits are rearranged to form a new number, is this new number also divisible by 11? If yes, give an explanation. If no, give a counterexample.

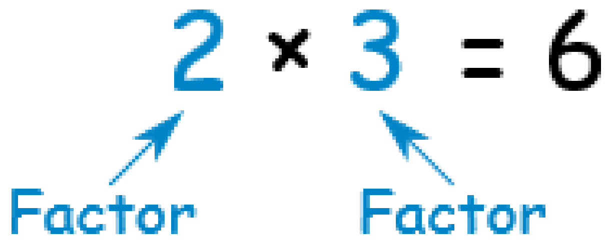
- Suppose that number is divisible by 3 and 9. If the digits are rearranged to form a new number, is this new number also divisible by 3 and 9? If yes, give an explanation. If no, give a counterexample

- The letters a and b are digits (i.e., 0, 1,…,9). Find an a and b so that 41a2330b0 is divisible by 8, but not divisible by 3.

- Find three pairs of digits a,b so that the number 5a31411b2 is divisible by 6, but is not divisible by 9. Explain your reasoning.

Prime Factorization

- A Prime Number can be divided evenly **only** by 1 or itself. And it must be a whole number greater than 1.
  - The first few prime numbers are: 2, 3, 5, 7, 11, 13, and 17
- "Factors" are the numbers you multiply together to get another number:



"Prime Factorization" is finding **which prime numbers** multiply together to make the original number.

- **What are the prime factors of 12 ?**
- It is best to start working from the smallest prime number, which is 2, so let's check:

  12 ÷ 2 = 6

- Yes, it divided evenly by 2. We have taken the first step!
- But 6 is not a prime number, so we need to go further. Let's try 2 again:

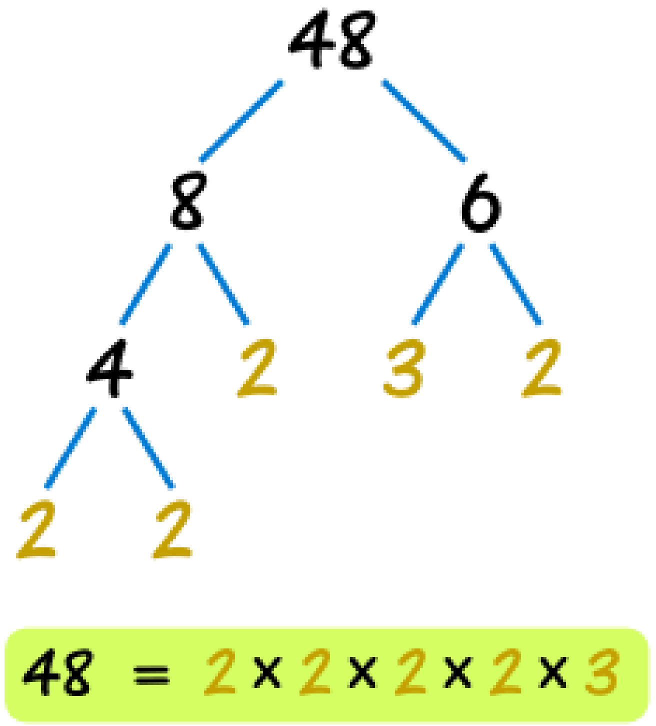
  6 ÷ 2 = 3

- Yes, that worked also. And 3 **is** a prime number, so we have the answer:

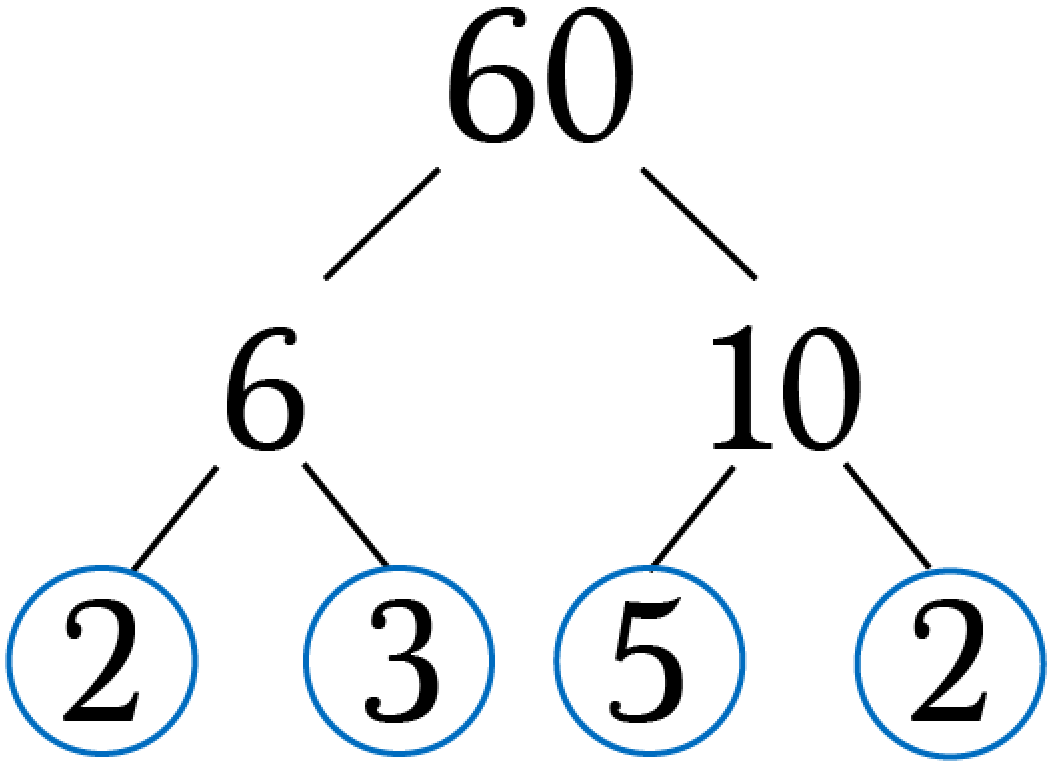
  12 = 2 × 2 × 3

"Factor Tree" can help: find **any factors** of the number, then the factors of those numbers, etc, until we can't factor any more

- **Example: 48**
- **48 = 8 × 6**, so we write down "8" and "6" below 48
- Now we continue and factor 8 into **4 × 2**
- Then 4 into **2 × 2**
- And lastly 6 into **3 × 2**
- We can't factor any more, so we have found the prime factors.
- Which reveals that **48 = 2 × 2 × 2 × 2 × 3**
- (or **48 = 2^4 × 3** using exponents)



How many factors?

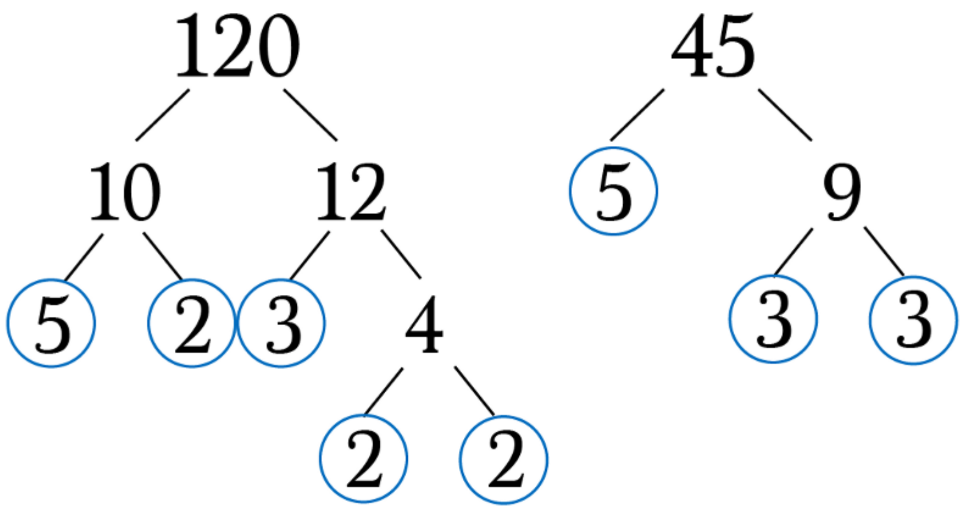


How many factors?

- 3640=

Finding the Greatest Common Factor

- The **GCF** is the largest number that divides into both values without a remainder. Let's find the **GCF of 120 and 45.**



- **Step 2:** Write out the prime factorizations for each.

$$2^3 \cdot 3 \cdot 5 \text{ and } 3^2 \cdot 5$$

- **Step 3**: The **GCF will be the prime factors that are common to both factorizations multiplied together**. In this example, both factorizations have one 3 and one 5, therefore the GCF is 3 x 5 or 15.

$$\text{gcf}(120, 45) = 3 \cdot 5 = 15$$

- *Note: The Greatest Common Factor and Greatest Common Divisor are exchangeable terms*.

- GCF for 48 and 90

The **LCM, least common multiple,** is smallest value that two or more numbers multiply into. Let's find the LCM of 120 and 45.

- Begin by using factor trees to write out each number's prime factorization. We have already found the prime factorizations for 120 and 45:

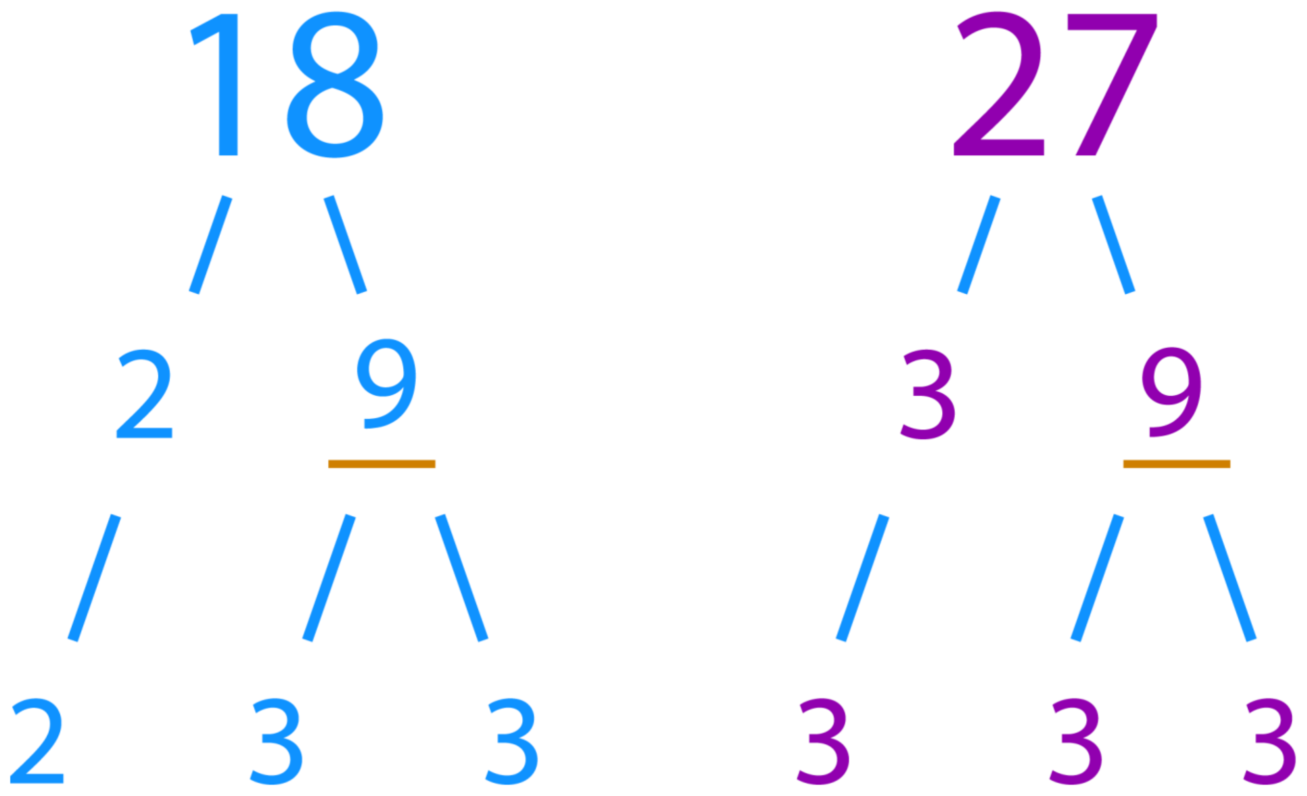
$$2^3 \cdot 3 \cdot 5 \text{ and } 3^2 \cdot 5$$

- The **LCM will be the product of the largest multiple of each prime that appears on at least one list.** For example we have a 2, 3 and 5, so I'll choose the largest multiples of each and find their product.

$$2^3 \cdot 3^2 \cdot 5 = 8 \cdot 9 \cdot 5 = 360$$

- Therefore the least common multiple of 120 and 45 is 360.

What is the highest common factor and the lowest common multiple between 18 and 27



- LCM 48 and 90

- Use the prime factorizations to find GCF and LCM of 28, 49, and 63.

- Use prime factorizations to compute the GCF and LCM of (42,63,210)

- Find GCF(2772,2940) and LCM(2772,2940)

- Use the prime factorizations to find GCF and LCM of 28, 49, and 63.

- Use prime factorizations to compute the GCF and LCM of (42,63,210)

- Find GCF(2772,2940) and LCM(2772,2940)

- At a summer camp, chocolate milk is served every other day, corn is served every 4 days and pizza every 7 days. Today all three were served. What is the smallest number of days until all three are served again?

- The organizers of a gymnastics event wish to arrange the participants in neat rows. They try rows of 2, 3, 4, 5, 6, 7 and 8, but in each case there is one gymnast left over. There are fewer than 1000 gymnasts in all. How many are there? Explain your reasoning. (Hint: What if 1 gymnast left the room?)

Trees

Graphs

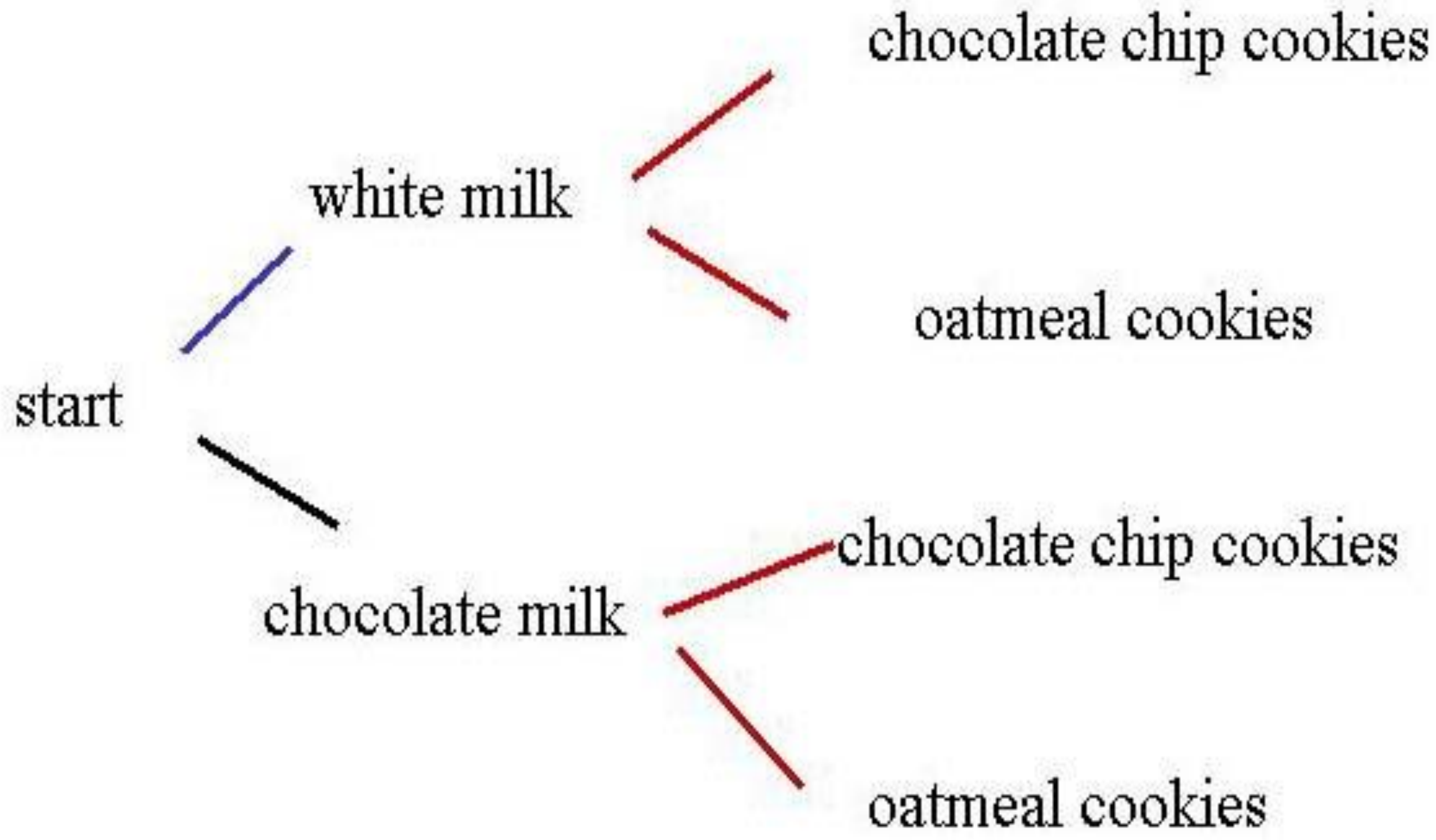
- More exactly, a graph consists of a set of n*odes* (also known as *points*, or *vertices*) with some pairs of these (not necessarily all pairs) connected by *edges*.
- The set of nodes of a graph *G* is usually denoted by *V* ; the set of edges, by *E*. Thus we write *G* = (*V,E*) to indicate that the graph *G* has node set *V* and edge set *E*.

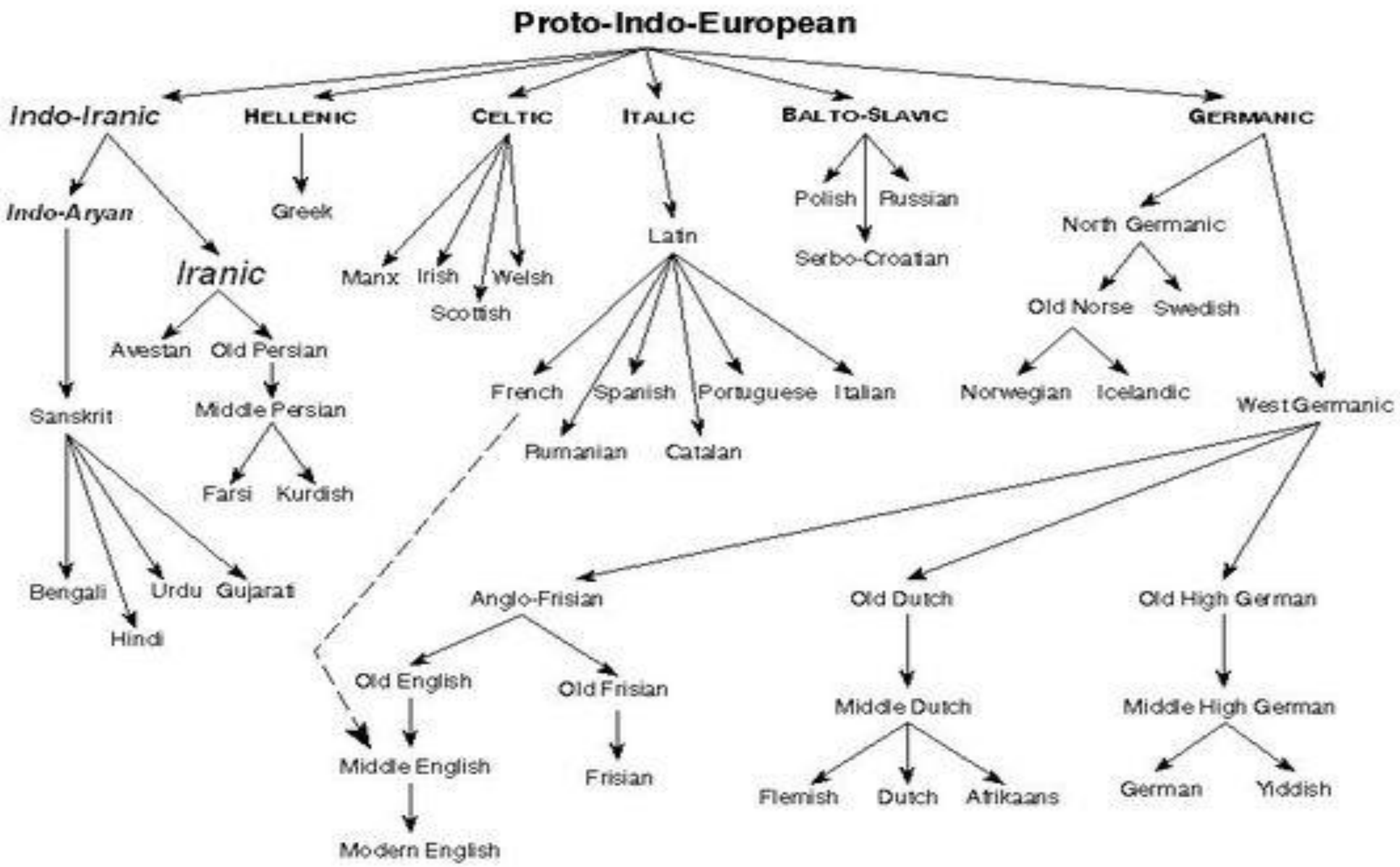
Trees

- A graph G = (V,E) is called a *tree* if it is connected and contains no cycle as a subgraph.
- *A graph G is a tree if and only if it is connected, but deleting any of its edges results in a disconnected graph.*

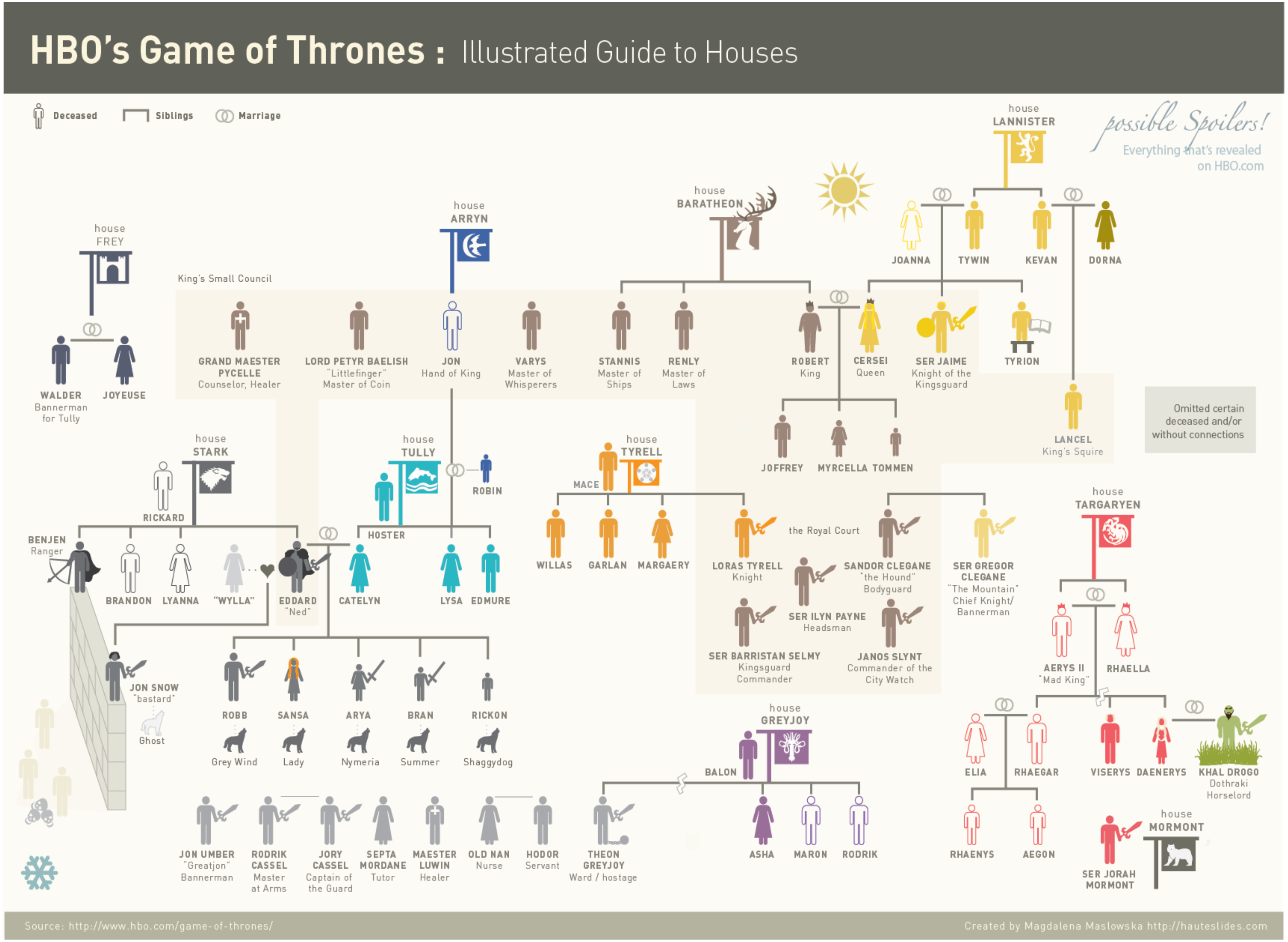
What is a Tree Diagram?

A tree diagram is simply a way of representing a sequence of events. Tree diagrams are particularly useful in probability since they record all possible outcomes in a clear and uncomplicated manner.


start
white milk
chocolate chip cookies
oatmeal cookies
chocolate milk
chocolate chip cookies
oatmeal cookies

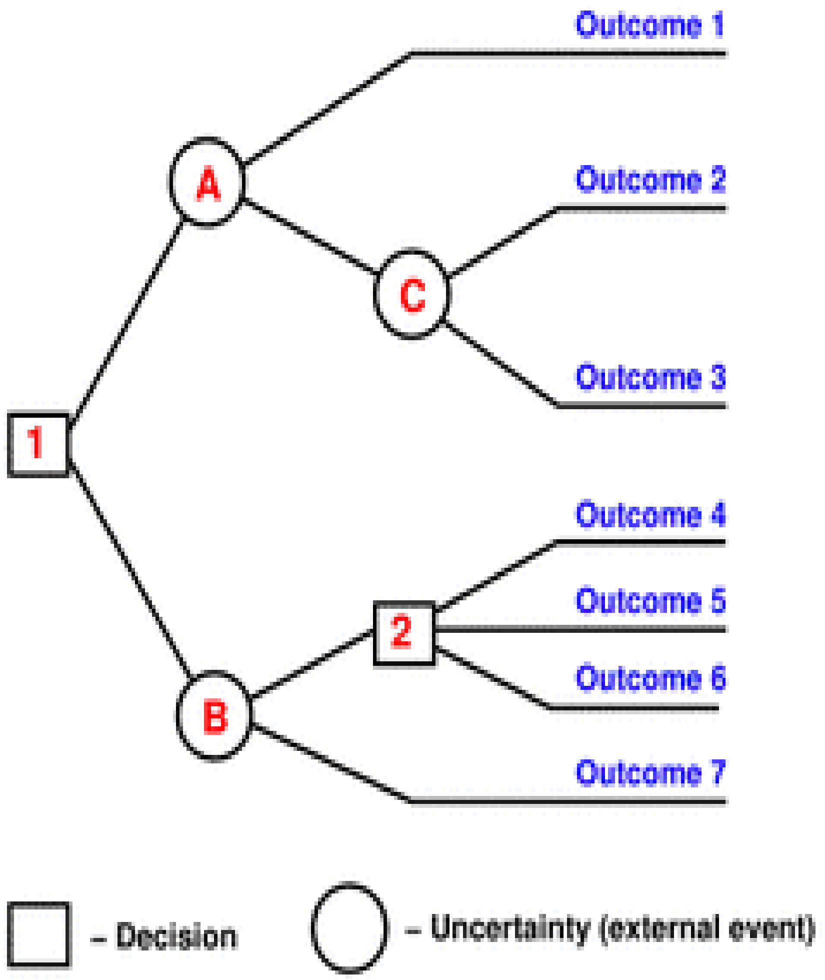


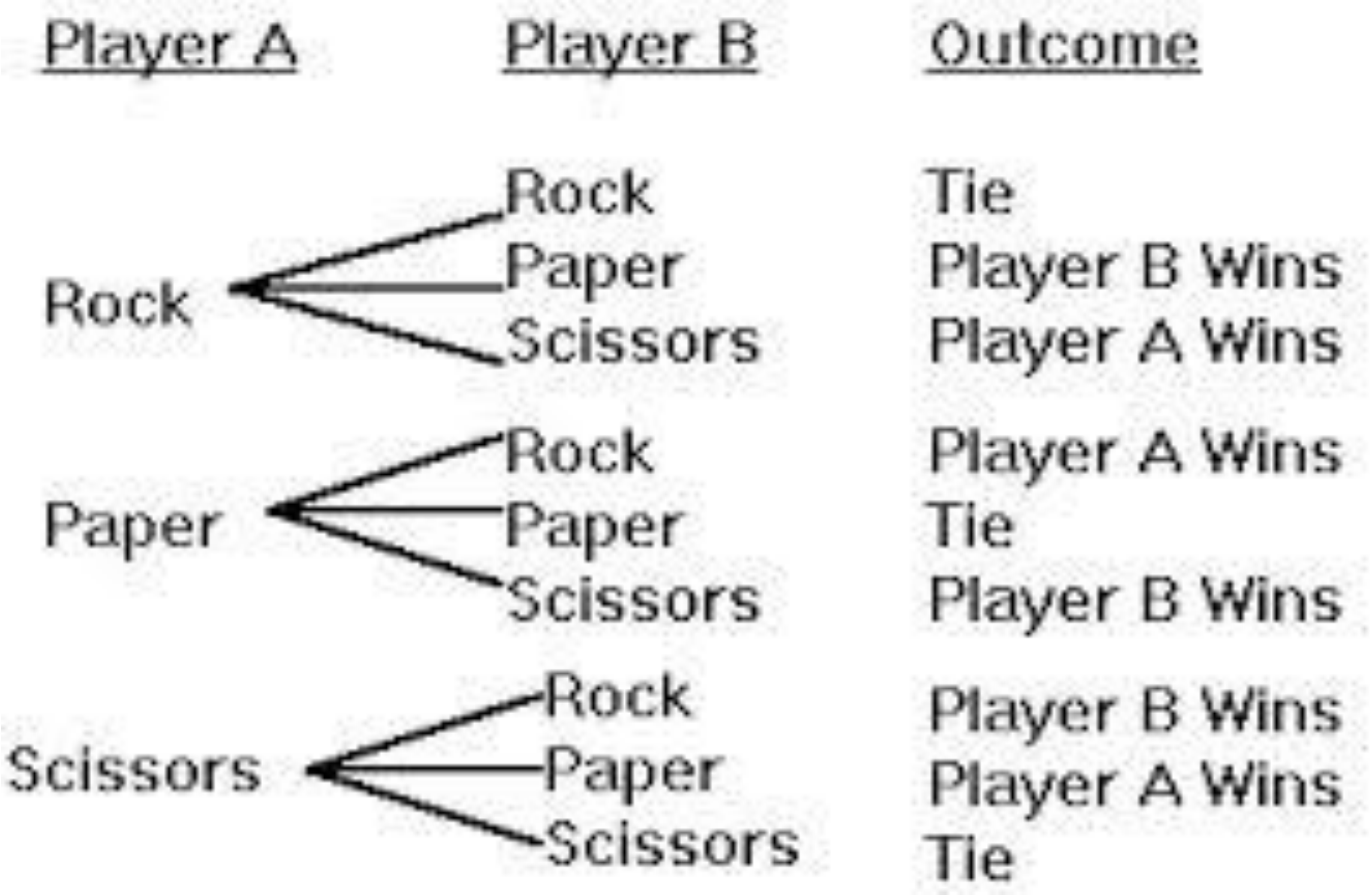
Proto-Indo-European
Indo-Iranic
Hellenic
Celtic
Italic
Balto-Slavic
Germanic
Indo-Aryan
Iranic
Greek
Manx
Irish
Welsh
Scottish
Latin
Polish
Russian
Serbo-Croatian
North Germanic
Old Norse
Swedish
Norwegian
Icelandic
West Germanic
Avestan
Old Persian
Middle Persian
Farsi
Kurdish
Sanskrit
Bengali
Urdu
Gujarati
Hindi
French
Spanish
Portuguese
Italian
Rumanian
Catalan
Anglo-Frisian
Old Dutch
Old High German
Old English
Old Frisian
Middle English
Frisian
Modern English
Middle Dutch
Flemish
Dutch
Afrikaans
Middle High German
German
Yiddish


HBO's Game of Thrones : Illustrated Guide to Houses
Deceased
Siblings
Marriage
possible Spoilers!
Everything that's revealed on HBO.com
house FREY
WALDER
Bannerman for Tully
JOYEUSE
King's Small Council
GRAND MAESTER PYCELLE
Counselor, Healer
LORD PETYR BAELISH
"Littlefinger"
Master of Coin
house ARRYN
JON
Hand of King
VARYS
Master of Whisperers
house BARATHEON
STANNIS
Master of Ships
RENLY
Master of Laws
ROBERT
King
CERSEI
Queen
house LANNISTER
JOANNA
TYWIN
KEVAN
DORNA
SER JAIME
Knight of the Kingsguard
TYRION
LANCEL
King's Squire
JOFFREY
MYRCELLA
TOMMEN
Omitted certain deceased and/or without connections
house STARK
RICKARD
BENJEN
Ranger
BRANDON
LYANNA
"WYLLA"
EDDARD
"Ned"
house TULLY
HOSTER
ROBIN
CATELYN
LYSA
EDMURE
house TYRELL
MACE
WILLAS
GARLAN
MARGAERY
LORAS TYRELL
Knight
the Royal Court
SANDOR CLEGANE
"the Hound"
Bodyguard
SER GREGOR CLEGANE
"The Mountain"
Chief Knight/ Bannerman
SER ILYN PAYNE
Headsman
SER BARRISTAN SELMY
Kingsguard Commander
JANOS SLYNT
Commander of the City Watch
house TARGARYEN
AERYS II
"Mad King"
RHAELLA
JON SNOW
"bastard"
Ghost
ROBB
Grey Wind
SANSA
Lady
ARYA
Nymeria
BRAN
Summer
RICKON
Shaggydog
house GREYJOY
BALON
ASHA
MARON
RODRIK
ELIA
RHAEGAR
VISERYS
DAENERYS
KHAL DROGO
Dothraki Horselord
RHAENYS
AEGON
house MORMONT
SER JORAH MORMONT
JON UMBER
"Greatjon"
Bannerman
RODRIK CASSEL
Master at Arms
JORY CASSEL
Captain of the Guard
SEPTA MORDANE
Tutor
MAESTER LUWIN
Healer
OLD NAN
Nurse
HODOR
Servant
THEON GREYJOY
Ward / hostage
Source: http://www.hbo.com/game-of-thrones/
Created by Magdalena Maslowska http://hauteslides.com

Decision Tree

- Decision Trees help you choose between multiple outcomes/courses you might take. They are very visual and help the user understand the risks and rewards associated with each choice.

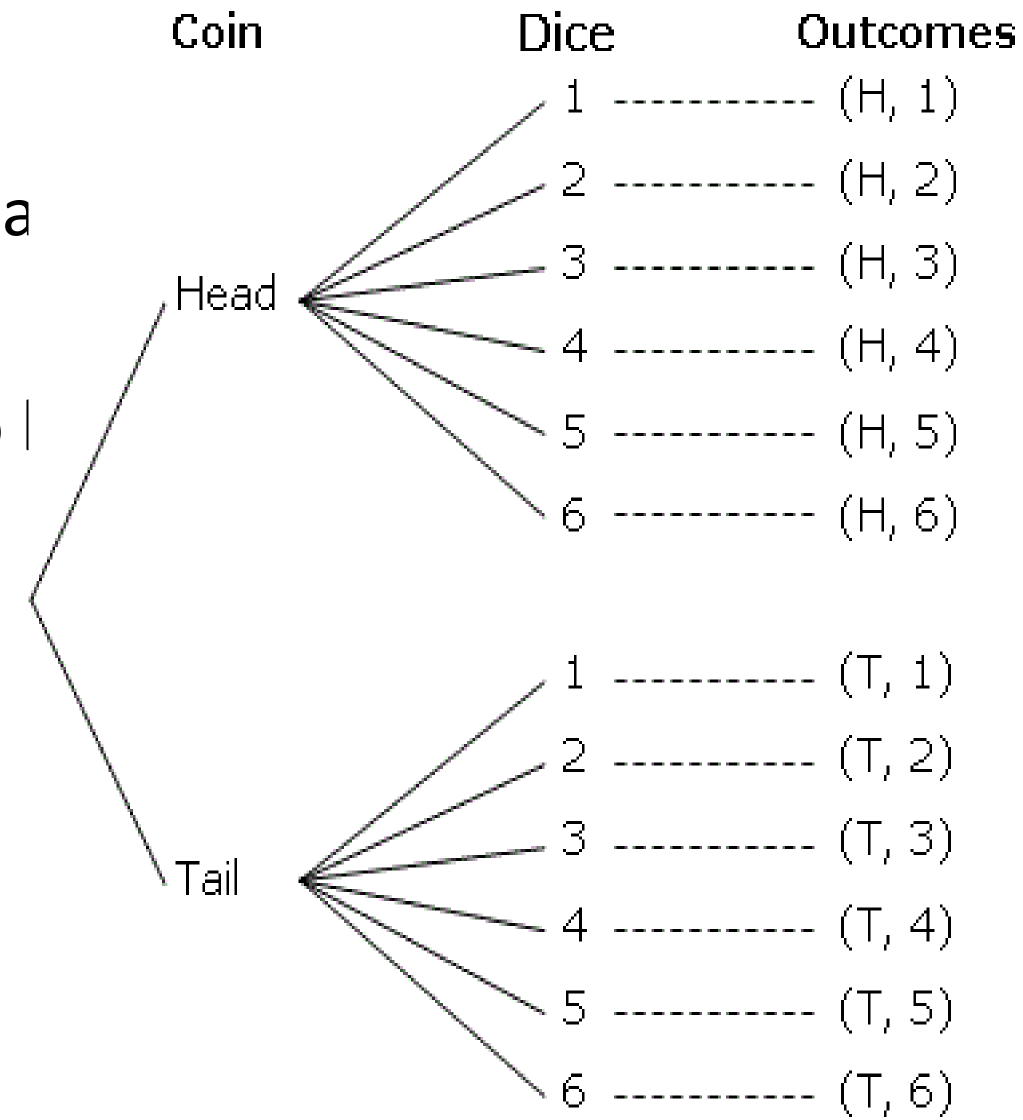



Player A
Player B
Outcome
Rock
Rock
Tie
Paper
Player B Wins
Scissors
Player A Wins
Paper
Rock
Player A Wins
Paper
Tie
Scissors
Player B Wins
Scissors
Rock
Player B Wins
Paper
Player A Wins
Scissors
Tie

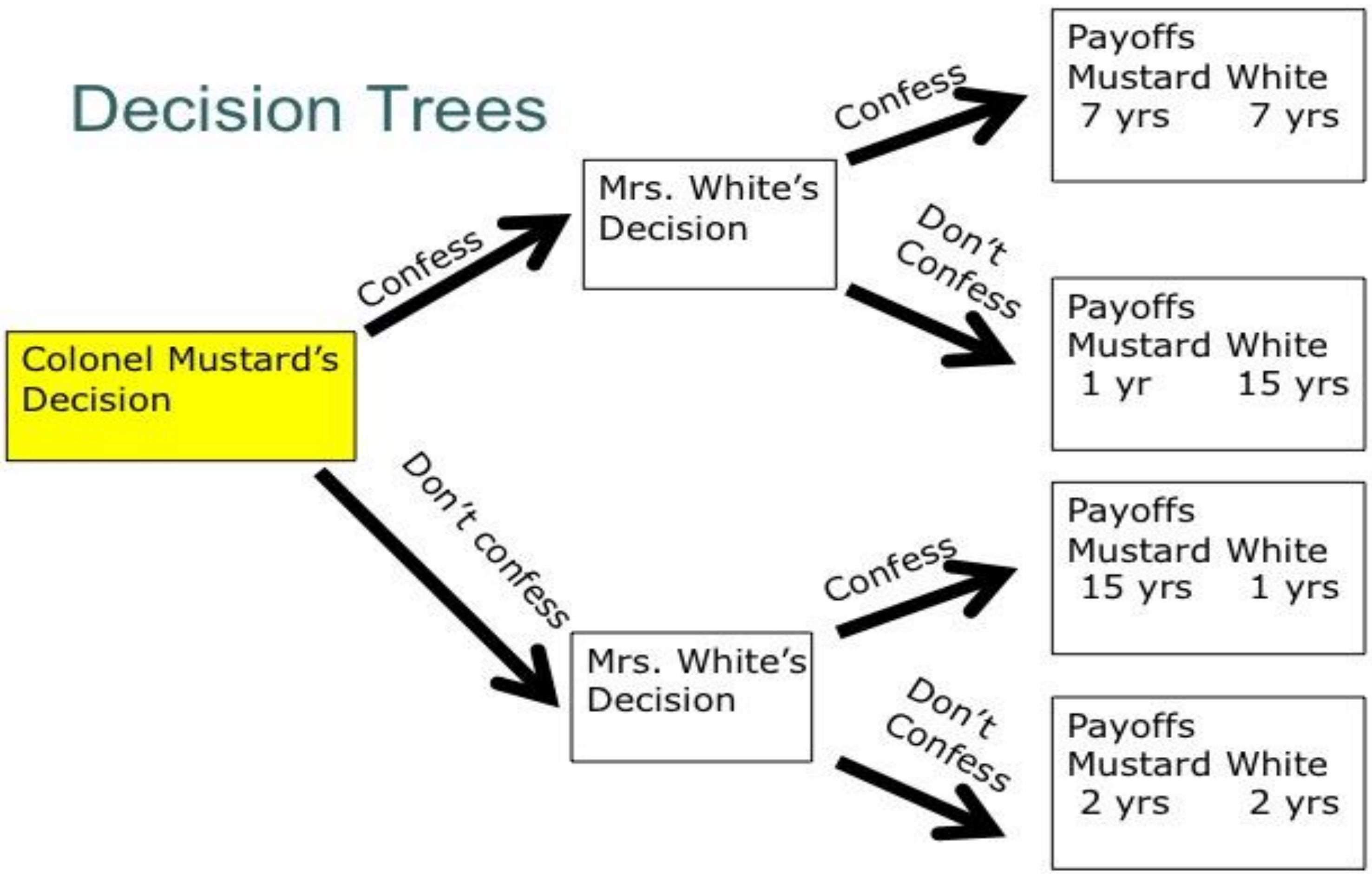
- A coin and a dice are thrown at random. Find the probability of:
- a) getting a head and an even number
- b) getting a head or tail and an odd number
- ***Solution:***
- We can use a tree diagram to help list all the possible outcomes.

- A coin and a dice are thrown a

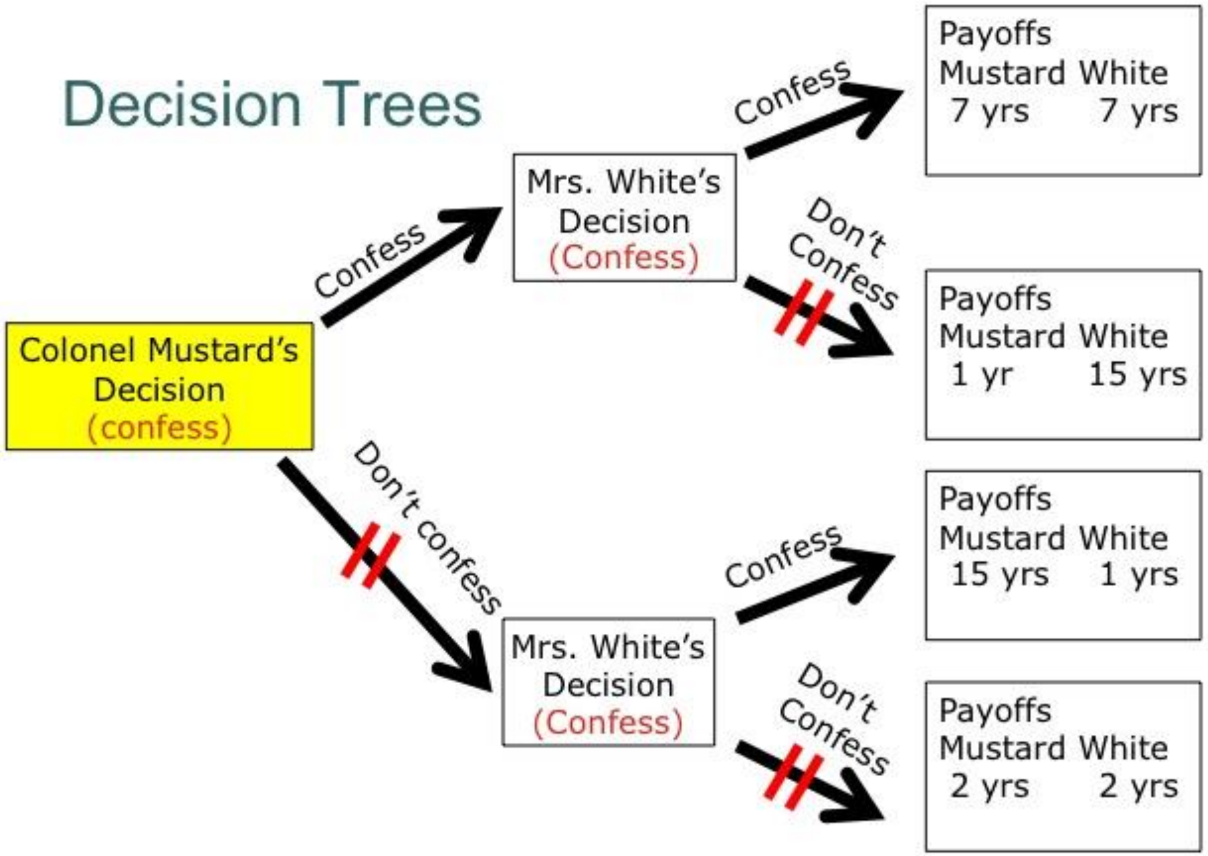
- We can use a tree diagram to



Decision Tree Sequential

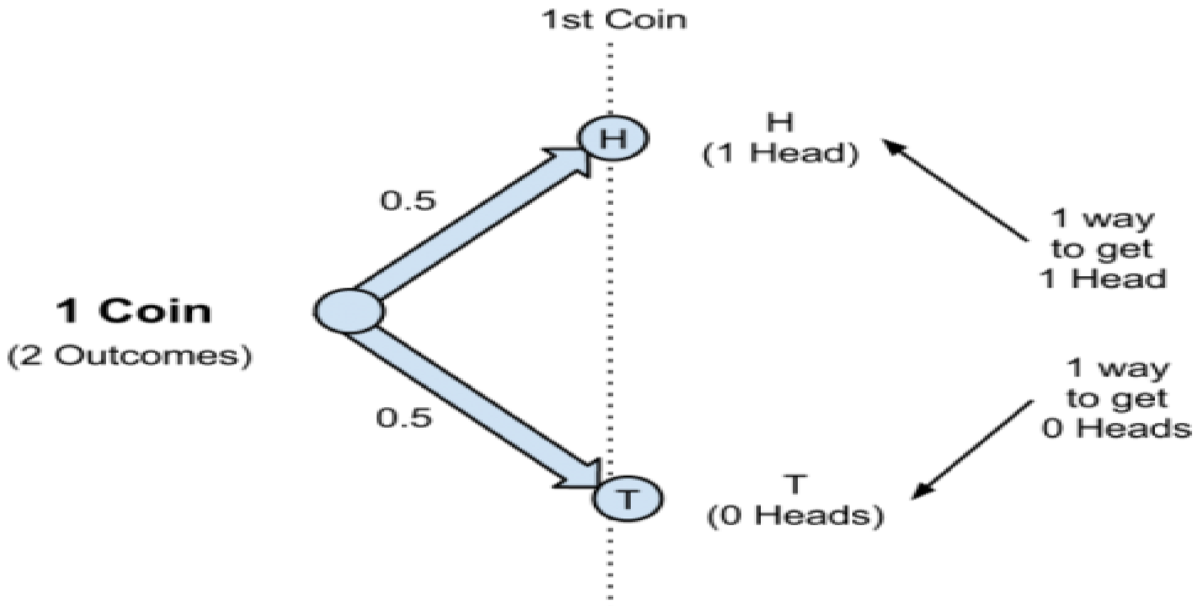


Solution: both confess



What Is Probability?

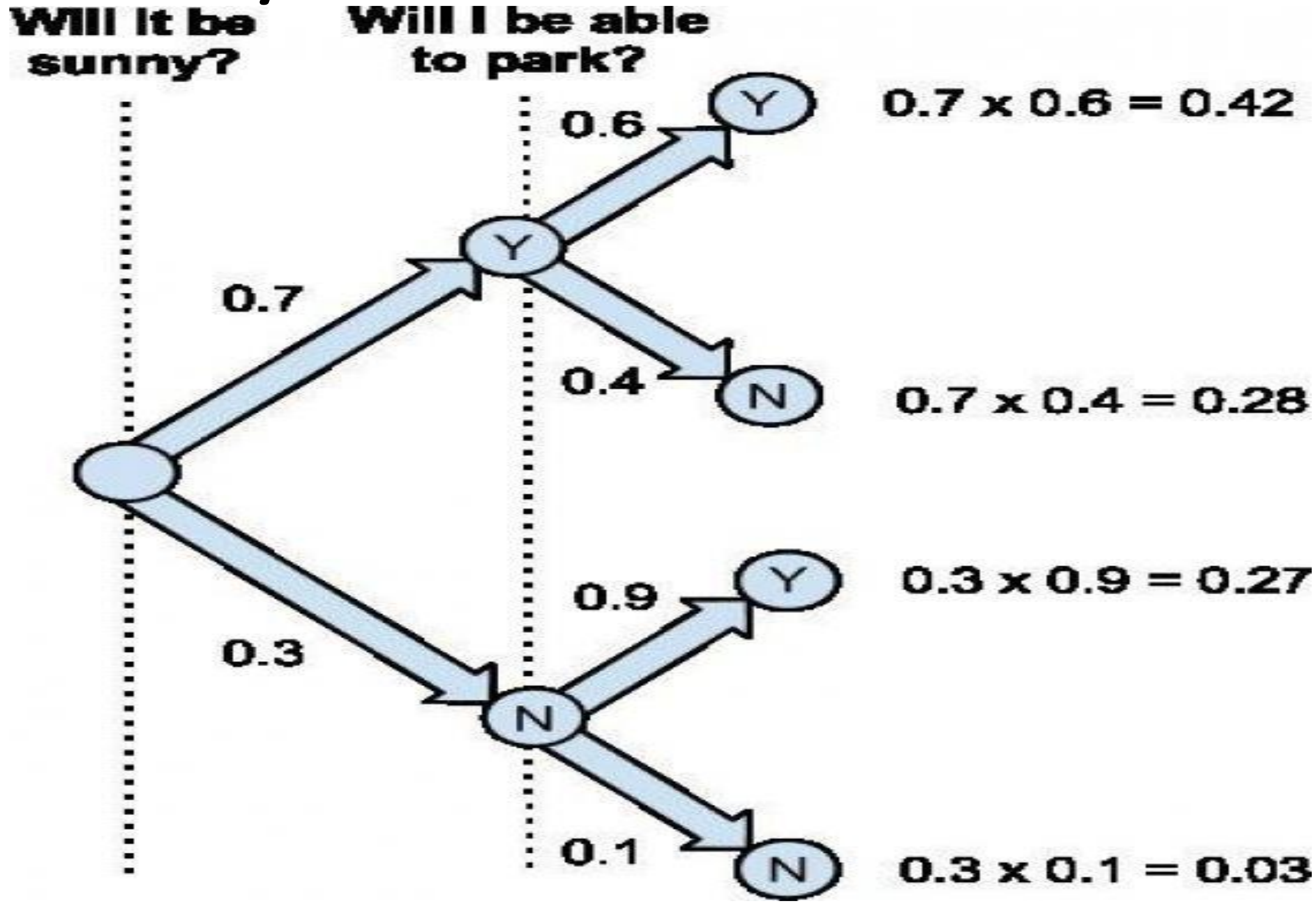
- Probabilities are decimal numbers or fractions between 0 and 1. The higher the probability (meaning the closer to 1), the more likely it is that whatever we're talking about will actually happen.
- When we toss a single coin there are exactly 2 possible outcomes—heads or tails—which we'll abbreviate as "H" or "T."

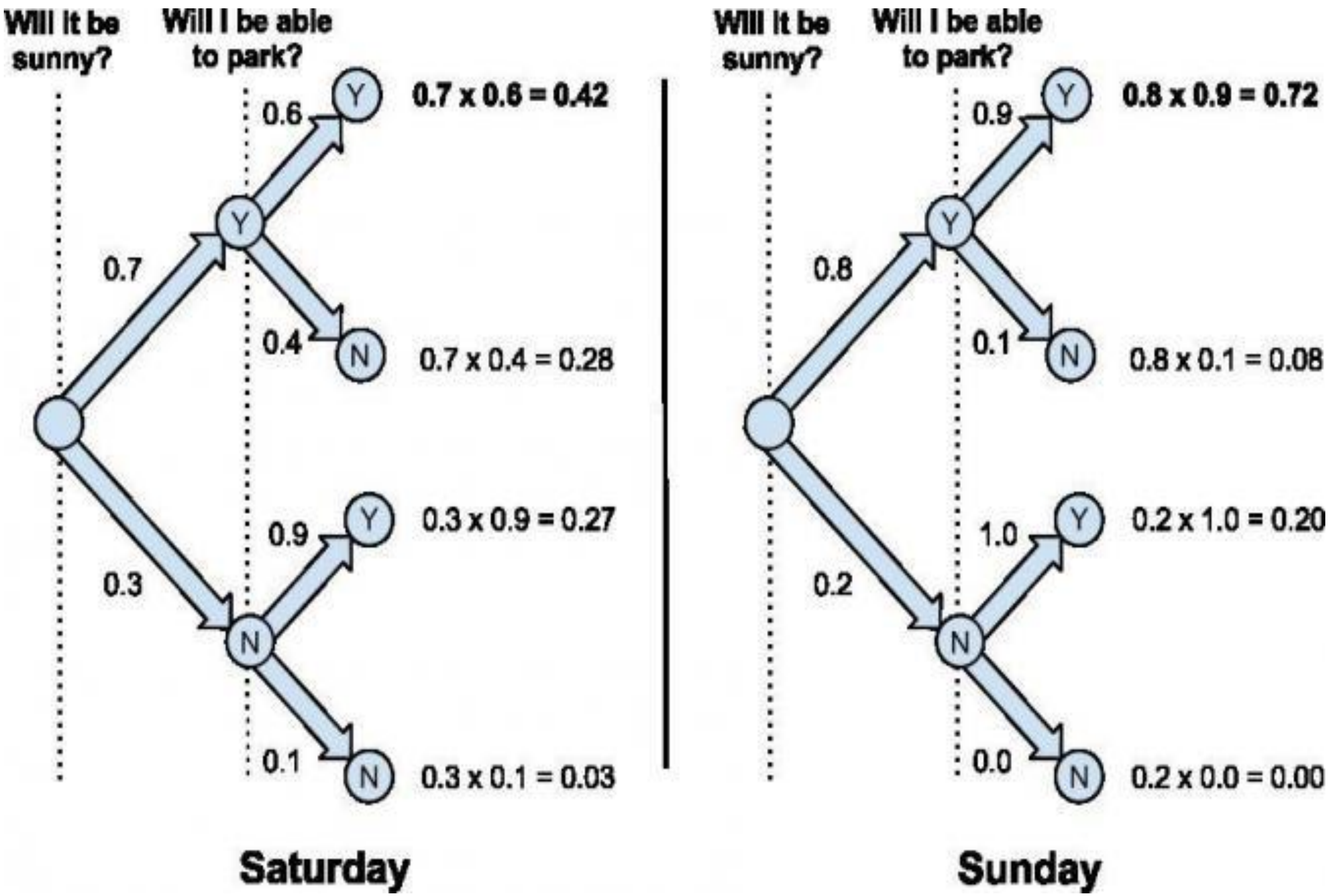


Probability Tree

- Many factors that influence your decisions can be translated into probabilities. Which means that we can use these factors to draw a probability tree to help us make decisions

A Probability Tree for the Beach





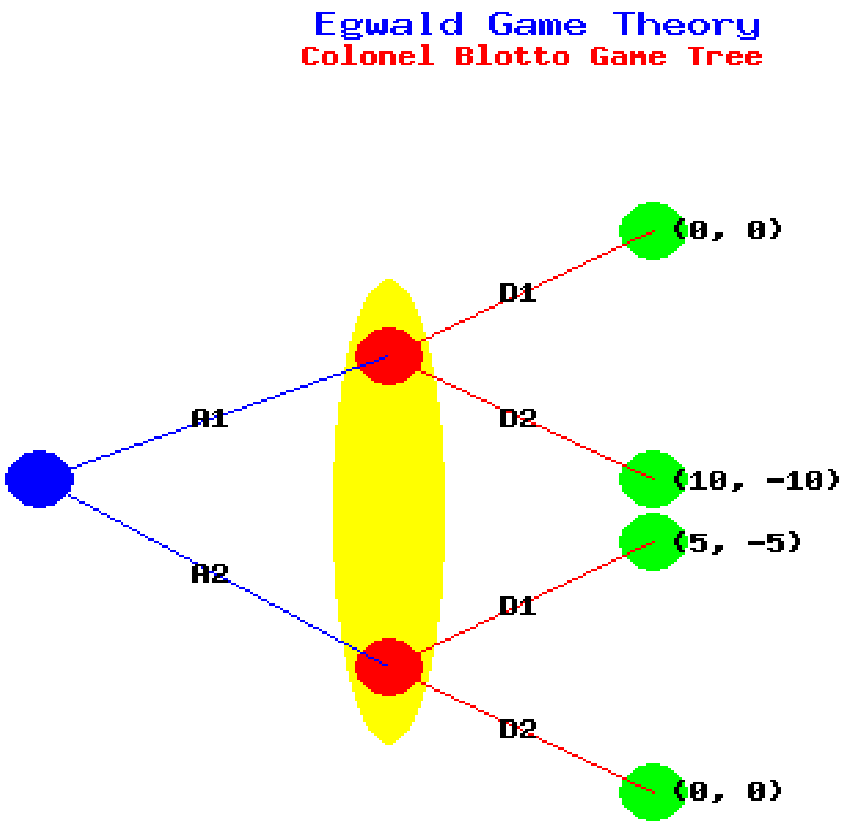
Will it be sunny?
Will I be able to park?
0.7
0.3
0.6
0.4
0.9
0.1
Y
N
0.7 x 0.6 = 0.42
0.7 x 0.4 = 0.28
0.3 x 0.9 = 0.27
0.3 x 0.1 = 0.03
Saturday
Will it be sunny?
Will I be able to park?
0.8
0.2
0.9
0.1
1.0
0.0
0.8 x 0.9 = 0.72
0.8 x 0.1 = 0.08
0.2 x 1.0 = 0.20
0.2 x 0.0 = 0.00
Sunday

- Colonel Sotto has two **"pure strategies,"** Attack City I and Attack City II; Colonel Blotto has two **"pure strategies,"** Defend City I and Defend City II. Each Colonel weighs each pure strategy against his enemy's pure strategies

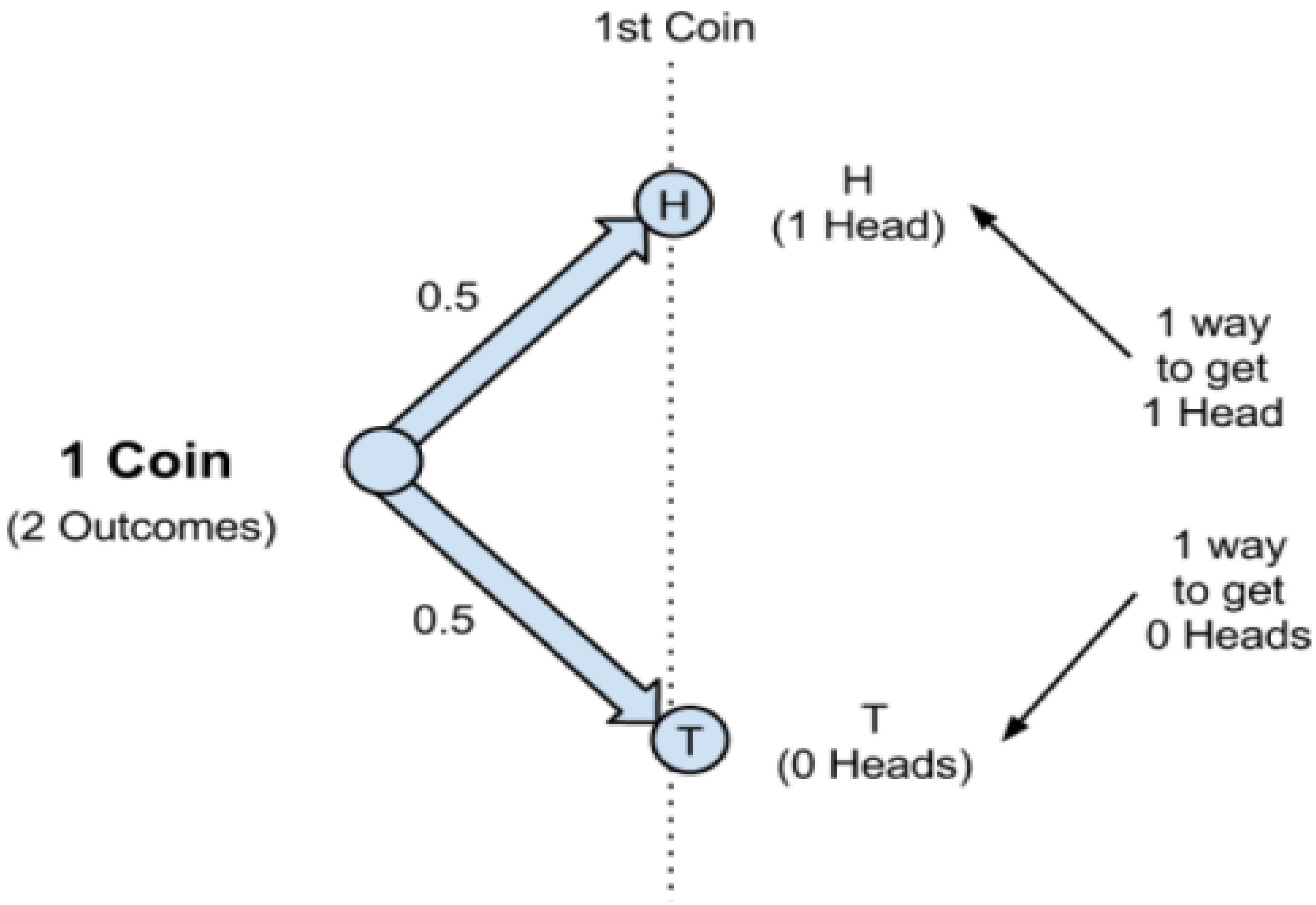
Attack and Defense Tableau		**Colonel Blotto**	
		Defend City I D1	Defend City II D2
Colonel Sotto	Attack City I A1	0	10
	Attack City II A2	5	0

- . Both probabilities for each Colonel must add-up to one. Therefore, the **optimal strategy for Colonel Blotto** is to defend City I with probability 2/3, and to defend City II with probability 1/3. In the meantime, the **optimal strategy for Colonel Sotto** is to attack City I with probability 1/3, and to attack City II with probability 2/3.

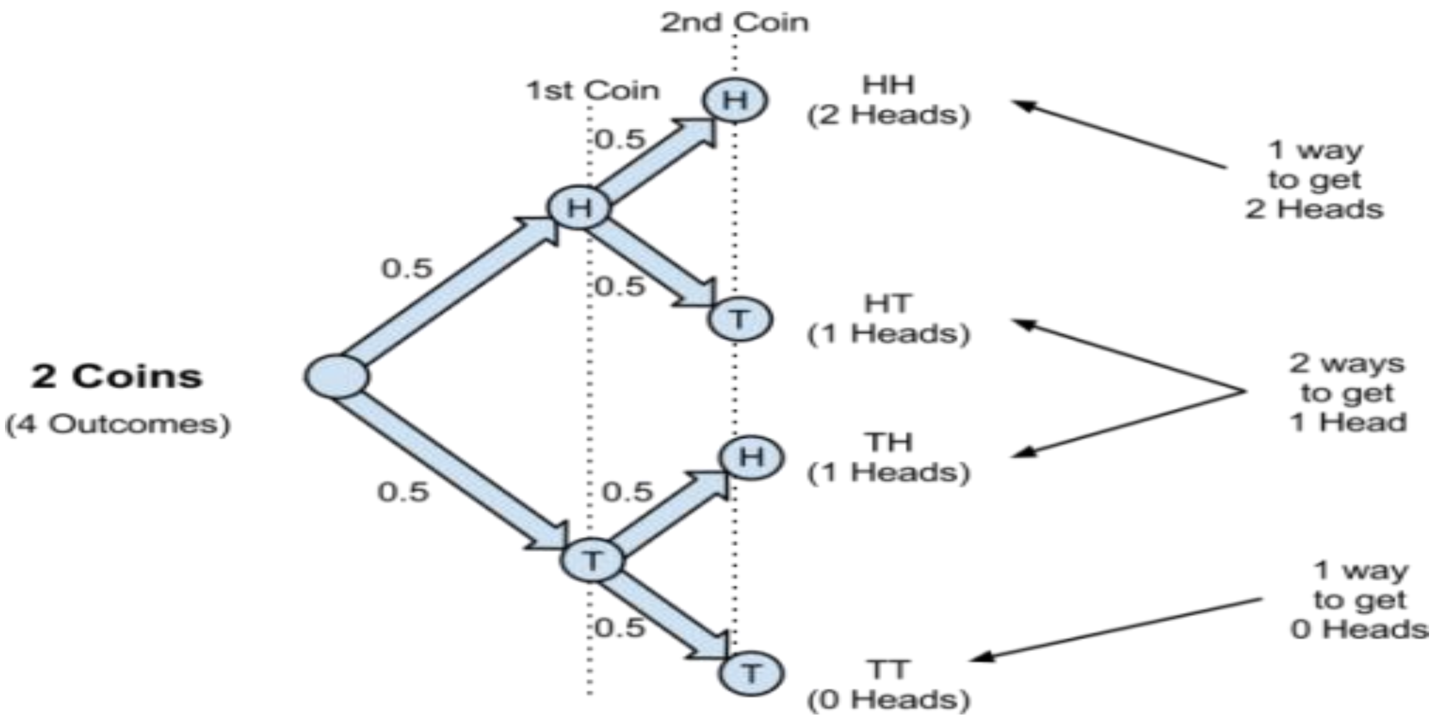
Decision Tree: simultaneous



Coin Problem

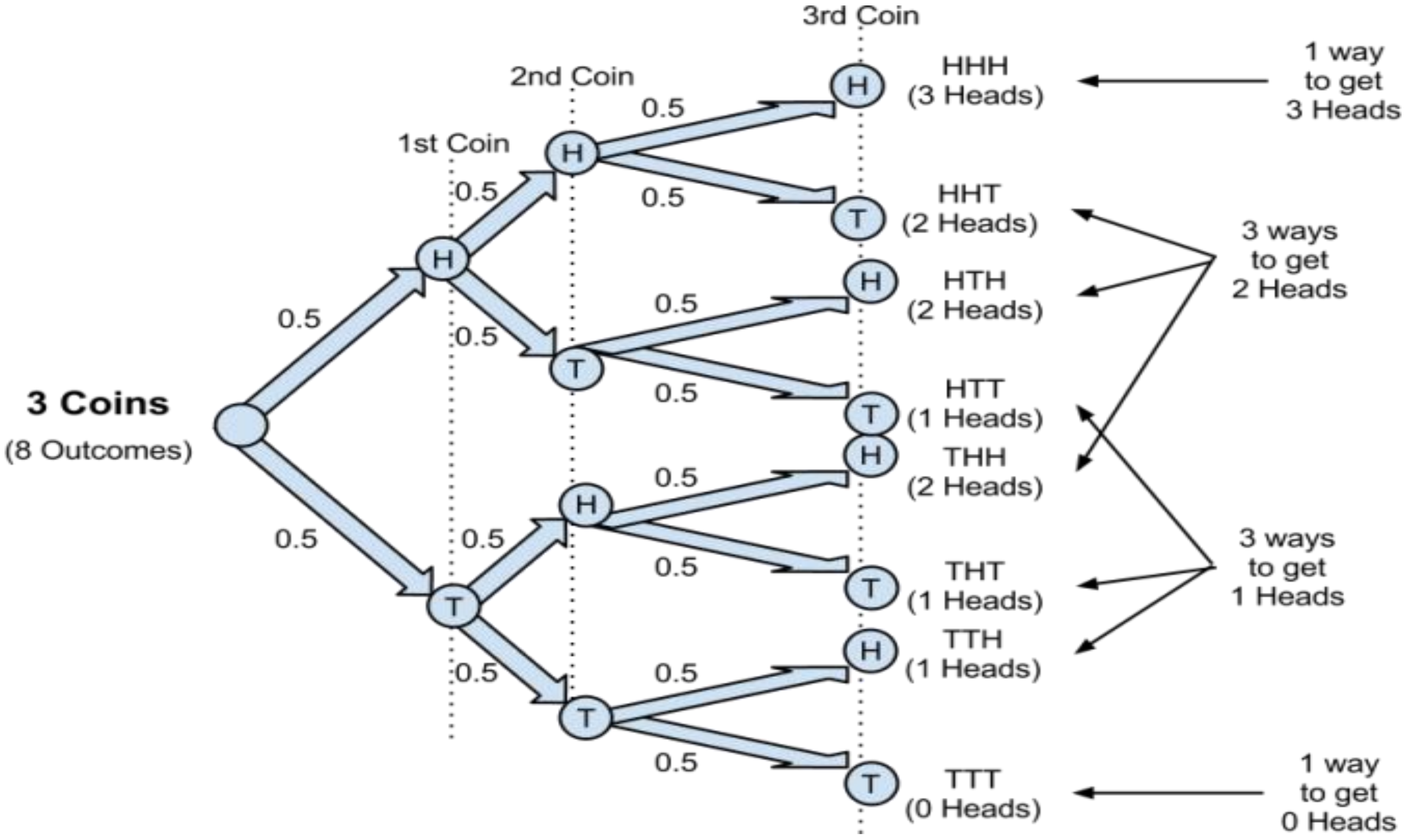


- Thinking about 1 coin is almost too easy, so let's move on to 2 coins

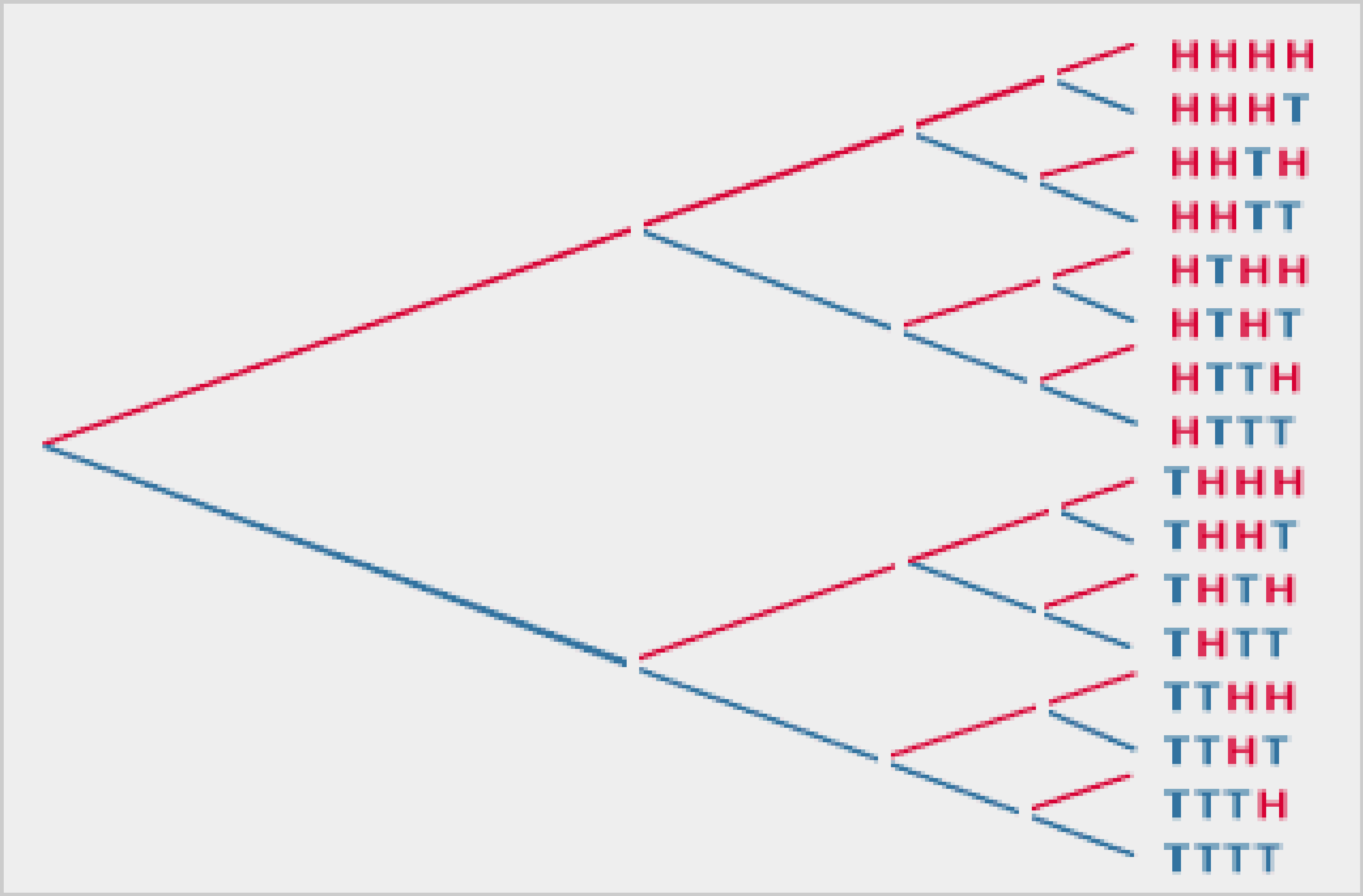


- There are 4 possible outcomes when tossing 2 coins. And there are! HH, HT, TH, or TT.

- Let's try tossing 3 coins at once.

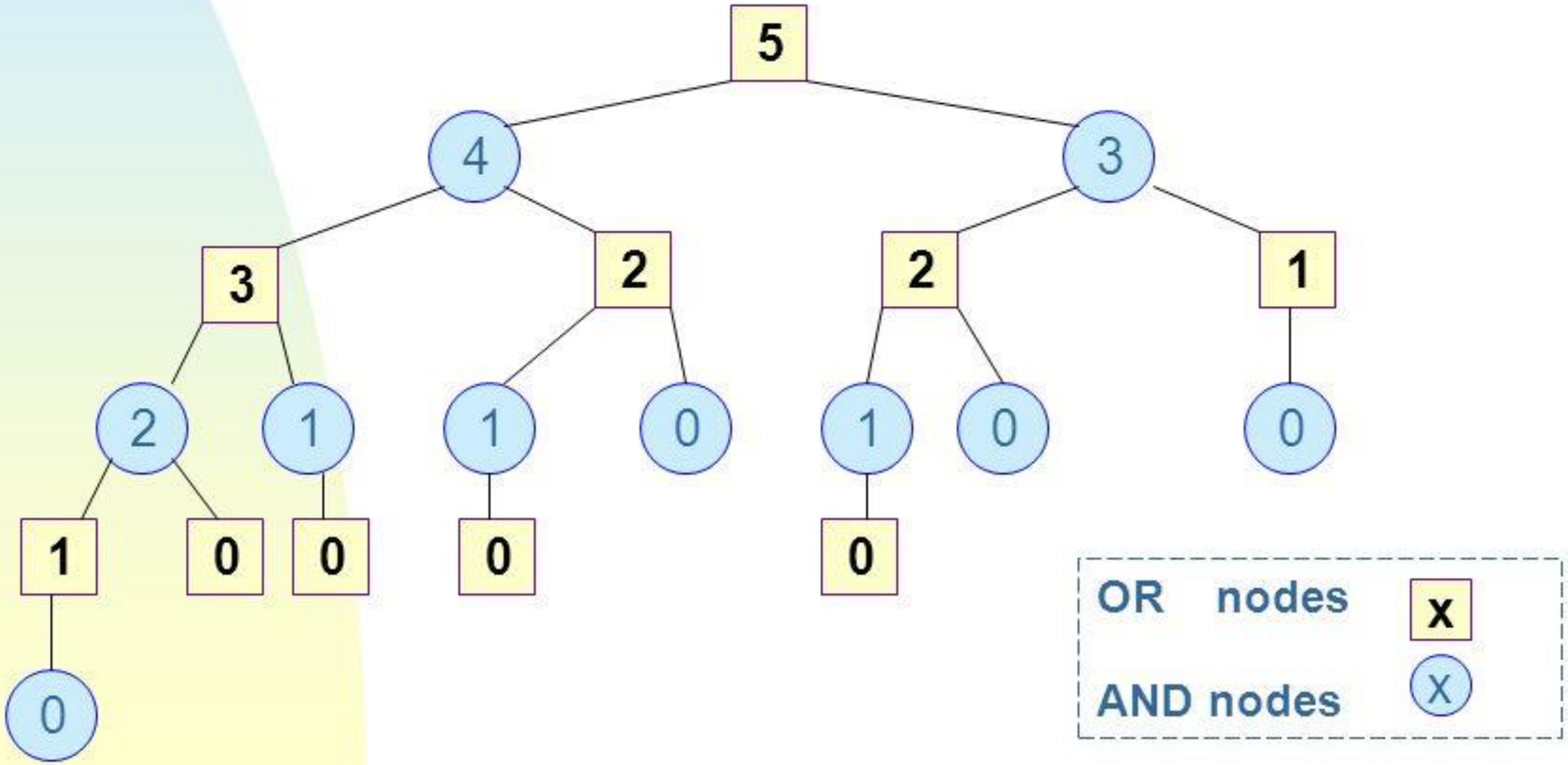


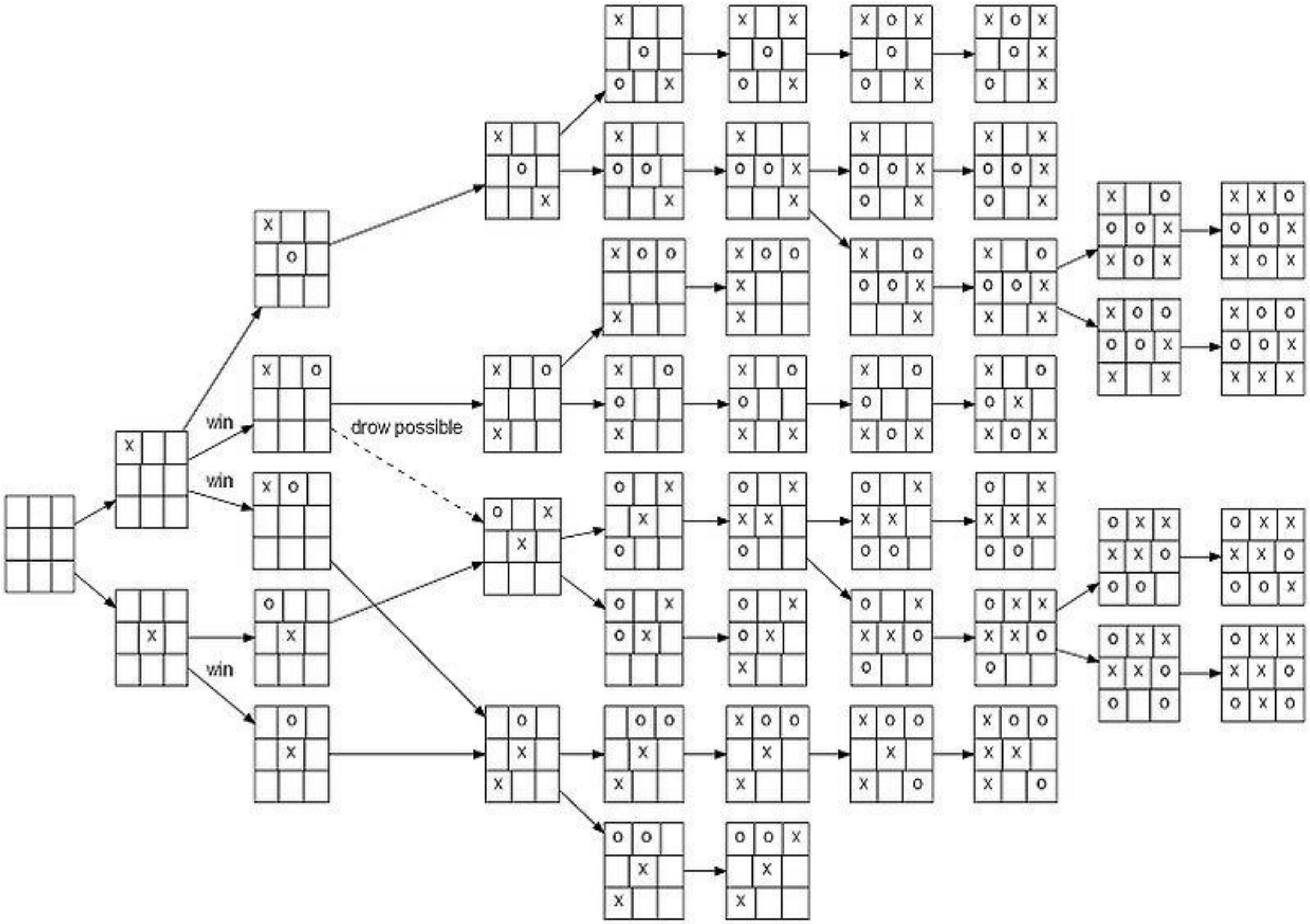
4 coin toss


HHHH
HHHT
HHTH
HHTT
HTHH
HTHT
HTTH
HTTT
THHH
THHT
THTH
THTT
TTHH
TTHT
TTTH
TTTT

Two-Player Game Trees

- The most common source of AND/OR graphs is 2-player perfect-information games.
- *Example*: Game Tree for 5-Stone Nim:




win
win
win
drow possible

Why are the divisibility test true? Use the division Lemma.

Proof of the test for 2:

By place value, any number n can be written

$$n = \underline{10a + b}$$

This # is $\underline{2(5a)}$, so it is div. by 2.

This is the test #, i.e., it is the last digit.

By the Division Lemma,

n is div. by 2 $\Longleftrightarrow$ b is div. by 2

$\Longleftrightarrow$ b = 0, 2, 4, 6, 8 □

- Example:

$$124 = \underbrace{12}_{a} \times 10 + \underbrace{4}_{b}\text{, and so forth.]}$$

Other proofs are similar:

1. Use place value to break # into the test case and a # which is div. by test #, and
2. Apply Division Lemma.]

- Proof for test for 4:

$$n = 100a + b$$

$100a = 4(25a)$; b: last 2 digits < 100

- Proof for test for 9:

$$n = 100a + 10b + c$$
$$= (99a + a) + (9b + b) + c$$
$$= (99a + 9b) + (a + b + c)$$

$99a + 9b = \underline{9(11a + b)}$; $a + b + c$: <u>sum of digits</u>

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