Workshop 2025/04/12
Introduction
Format of the competition (3 stages, number of questions, format of answer, scoring, winners, prizes, etc…)
Few important definitions:

Basic terminology:
Whole Number, (positive). Examples 1, 4, 11

Odd number, (either positive or negative): any number that is not divisible by 2. Examples 3,-5,17

Even number, (either positive, zero, or negative): any number that is divisible by 2. Examples -10,0,32

Prime number: any whole number greater than one that is only divisible by itself and by 1. Examples 2,3,17

The K-th power of a number N: The notation is N^K. It is defined as $N \times N \times N \cdots \times N$, (K times).

Some basic terminology of 2-D, (two dimensional), geometric shapes, (or objects):
A regular polygon: a 2-D shape whose boundaries are all straight lines, and all the angles between any 2 of these lines are the same.
Examples of regular polygons: equilateral triangle, square, regular pentagon, regular hexagon, regular octagon.

Some definitions of angles: **Acute**: $0 < \measuredangle < 90°$, **Right**: $\measuredangle = 90°$, **Obtuse**: $90° < \measuredangle < 180°$.
Sum of all the angles of a triangle: $180°$.
Sum of all the angles of a polygon: any polygon with N sides can be divided by straight lines into $N - 2$ triangles. Thus, the sum of all its angles in degrees is: $(N - 2) \times 180$.

Triangles:
A triangle is a 2-D shape made up of 3 straight lines.
Right triangle: one of the angles is $90°$.
Isosceles triangle: the length of two of the sides of the triangle is the same.
Equilateral triangle: the length of all the sides is the same.
Obtuse triangle: one of the angles of the triangle is more than $90°$.
Pythagorean theorem: every right triangle satisfies $a^2 + b^2 = c^2$, where a, b, c are its sides (c is the largest side of the triangle).

Some definitions of non-regular polygons with 4-sides:
Rectangle: all angles of polygon are right angles.
Parallelogram: 2 pairs of parallel sides.
Rhombus: the length of all sides is the same.

A Circle is defined as a round 2-D shape. The **Radius** of the circle is a straight line connecting its centre with its boundary. The **Diameter** of the circle is a straight line dividing the circle into 2 areas of equal size. Note that this line goes through the centre of the circle and its length is twice as large as the length of the radius.
The **Circumference** of the circle is the length of its rounded boundary. There is a certain constant named π that relates the circumference and the area of a circle to its radius, r. π is not a rational number, (see below for the definition of a rational number), and its value is approximately $\pi \approx 3.14 \cdots$.
Circumference, $C = 2\pi r$. Area, $A = \pi r^2$.
A Sector of a circle is a shape surrounded by 2 radii and portion of boundary of the circle.

Definitions and properties of simple 3-D objects that have 8 **Corners** such as a **Cube** or a **Box,** and whose all angles are right angles:
The 12 **Edges,** (the lines connecting 2 adjacent corners), are straight lines.
The 6 **Faces** are the 2-D shapes enclosed in 4 edges.
If x, y, z are the edges of a box, its total **Surface area** is $S = 2(xy + xz + yz)$ and its **Volume** is given by $V = xyz$.

A rational number, (or otherwise called a fraction) is a number that is either $\pm\frac{m}{n}$ where n is a, (positive), whole number and m is either zero or a, (positive), whole number.

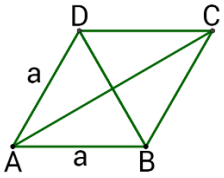
Division of a positive whole number p by another positive whole number q, i.e. $\frac{p}{q}$, may be a whole number, or may have a **Remainder**, $0 < r < q$. Note that p is called the **Numerator**, and q is called the **Denominator.** If this division has no remainder, then the value of $\frac{p}{q}$ is called the **Quotient.**

A Factor of a whole number n is another whole number m that divides n.
For any **Fraction** where the numerator and the denominator have no common factor, that fraction is called: "**Fraction lowest terms**."

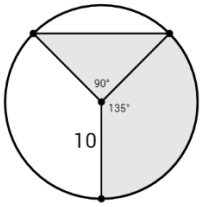
A Random event is an event with no certainty that it will happen. For example, if tossing a fair coin there is "about" a 50% chance that the toss will be "Head" and "about" 50% chance that the toss will be "Tail". The terminology used to describe the chances of events to occur is called the **Probability** of these events.
Thus, the probability P of any event satisfies $0 \leq P \leq 1$.
The probability of an event to occur may change if more information is given. The terminology used to define such probability is called: "**Conditional probability**".
For example: You roll 2 dice, and it is easy to show that the probability that the event "the sum of the 2 numbers showing is less than 5", (i.e. 2,3, or 4), is $\frac{6}{36} = \frac{1}{6}$, or to be more specific, 5 rolls, (out of total possible of 36 different rolls), satisfy the condition, namely: $(1,1), (1,2), (1,3), (2,1), (2,2), (3,1)$. If you know that the sum is also less than 12, (the additional information), then the roll of $(6,6)$ is excluded as a possible roll, so the total number of possible rolls is only 35, and the probability that the sum is less than 5 is now: $P = \frac{6}{35}$.

Worksheet #1

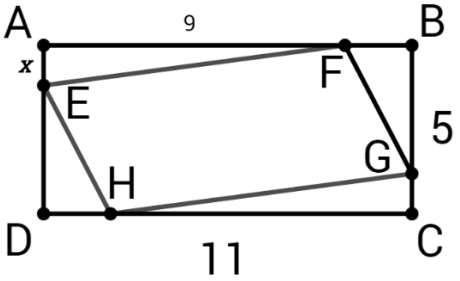
1. Below is a rhombus with sides $a = 7$. Its short diagonal satisfies $BD = a = 7$. What is the square value of the long diagonal, (i.e. the value of AC^2) ?



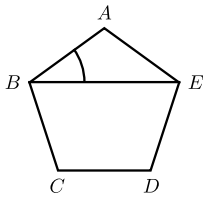
2. The shaded section of the circle of radius 10 consists of a right triangle and a sector of $135°$. Find the area of the shaded section. Use $\pi = 3.14$, and round your answer to the nearest whole number.



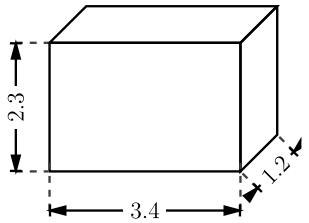
3. $ABCD$ is a rectangle with sides 11 and 5. $EFGH$ is a parallelogram. $AE = x$, $AF = 9$. The area of $EFGH$ is $\frac{2}{3}$ of the area of $ABCD$. What is the area of ΔDEH? Express the answer as a fraction in lowest terms.



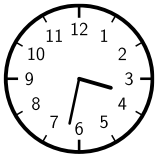
4. $ABCDE$ is a regular pentagon. What is the value of $\measuredangle ABE$ in degrees (°)?



5. What is the total area of the 6 faces of a box with sides 1.2, 2.3, and 3.4? Round the answer to the nearest whole number.



6. What is the value, in degrees, (°), of the acute angle between the hour hand and the minute hand of a clock at the time of $3:32$ PM?



Worksheet #2

1. $\frac{1}{x}+\frac{1}{2x}+\frac{1}{3x}=3$. Express x as a fraction in lowest terms.

2. In how many ways can you pay 80 cents using any combination of 5, 10, and 25 cent coins?

3. A triangle has sides L, M, and N, where $0<L<M<N<12$ are all whole numbers. The perimeter of the triangle is P. How many different values of P are there?

4. There is a pile of 5 cards numbered $1,2,3,4,5$ on the table. Gloria takes 3 different cards at random from the pile and writes down the sum of these 3 cards. What is the probability that the sum is a multiple of 3? Express the answer as a fraction in lowest terms.

5. x is a 2-digit positive number whose digits are two consecutive odd numbers.
   y is defined to be the 2-digit positive number with the digits of x reversed.
   What is the maximum possible value of $N=x+y$?

Worksheet #3

1. Eric takes 4 times longer to paint a ceiling than to paint a wall. He charges 20% more per hour to paint a ceiling than to paint a wall. Eric painted 5 ceilings and 12 walls. His hourly charge per wall was $40 and his total earning was $2520. How many hours did he work in total?

2. The following is an arithmetic sequence: $N, N+K, N+2K, N+3K, \cdots$. The value of the 2024-th term of the sequence is 10000, and the value of the 1000-th term is 784 . What is the value of N (the first term)?

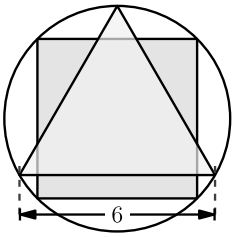
3. Express $0.5333\cdots$ as a fraction in lowest terms.

4. A box contains 5 white marbles and 8 black marbles. Andrew took 2 marbles out of the box at random and placed them on the table. What is the probability that at least 1 of the marbles was white? Express the answer as a fraction in lowest terms.

5. Of a deck of 9 cards, 4 cards are aces. Dan was dealt, (at random), 3 cards, and at least one of the cards that was dealt to him was an ace. What is the probability that he was dealt at least 2 aces? Express the answer as a fraction in lowest terms.

Worksheet #4

1. An equilateral triangle and a square are inscribed in the same circle.
The length of the side of the triangle is 6. Calculate the area of the square.



2. A box with whole number sides L, M, and N has volume $V = 3059$.
Another box with the same volume, V, has sides $L + 4$, $M - 4$, and N.
What is the value of $L + M + N$?

3. How many pairs (N, M) are there such that $N > 0$, $M > 0$ are whole numbers, and $3N + 7M = 100$?

4. What is the largest even 9-digit number that is divisible by 9 and its digit sum is 63 ?

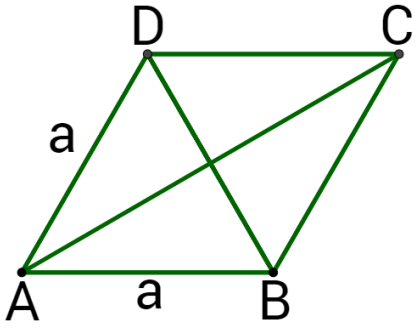
5. Three dice are rolled. What is the probability that the sum of the dice is less than 8 ?
Express the answer as a fraction in lowest terms.

6. One of the water tanks of the Vancouver Aquarium has two taps. The first tap fills it at a rate of 15 litres per minute and the second tap at 20 litres per minute. Both taps were turned on when the tank was empty. But, the drain pipe was partly open and water was leaking out of the tank at a rate of 6 litres per minute during the first 2 hours before it was closed. The tank was finally filled up in 6 hours.
What is the volume of the tank in m^3, (cubic metres)?
Express the answer as decimal correct to two decimal places.

Worksheets with solutions

Worksheet #1

1. Below is a rhombus with sides $a = 7$. Its short diagonal satisfies $BD = a = 7$. What is the square value of the long diagonal, (i.e. the value of AC^2) ?



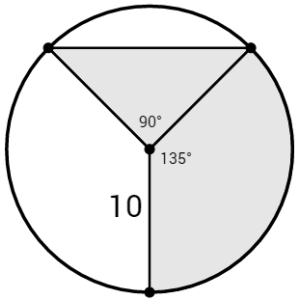
1. Triangles ΔABD and ΔBCD are congruent. Thus, in a rhombus BD is perpendicular to AC. Also, it is clear that the 2 diagonals are bisected.

Using the Pythagorean Theorem $(\frac{AC}{2})^2 + (\frac{a}{2})^2 = a^2$.

So, $(\frac{AC}{2})^2 = a^2 - \left(\frac{a}{2}\right)^2 = \frac{3}{4}a^2 = \frac{3}{4} \times 49 = \frac{147}{4}$.

So, $AC^2 = 4 \times \frac{147}{4}$=147.

2. The shaded section of the circle of radius 10 consists of a right triangle and a sector of $135°$. Find the area of the shaded section. Use $\pi = 3.14$, and round your answer to the nearest whole number.



2. The shaded area is consisted of a right triangle and a sector of the circle.

The area of the right triangle is:

$\frac{10 \times 10}{2} = 50$.

The area of the $135°$ sector is (using the approximation for π):

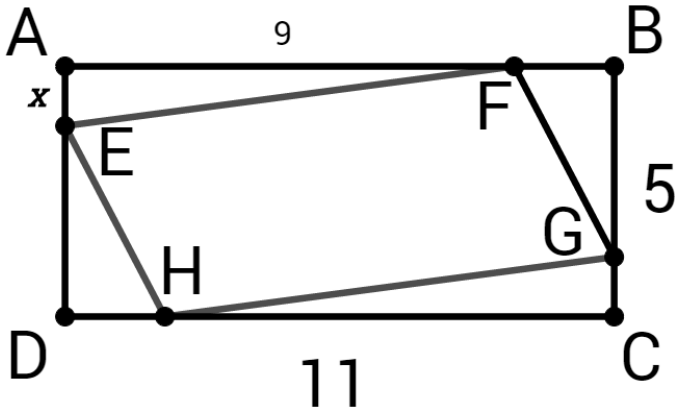
$3.14 \times 10^2 \times \frac{135}{360} = 314 \times \frac{3}{8} = 117.75$.

So, the area of shaded section is:

$50 + 117.75 = 167.75$.

Rounding the area to the nearest whole number the requested solution is: 168.

3. $ABCD$ is a rectangle with sides 11 and 5. $EFGH$ is a parallelogram. $AE = x$, $AF = 9$. The area of $EFGH$ is $\frac{2}{3}$ of the area of $ABCD$. What is the area of ΔDEH? Express the answer as a fraction in lowest terms.



3. $EFGH$ is a parallelogram. Note that EF is not necessarily perpendicular to EH.

Area of $ABCD$ is 55 so the sum of the areas of AEF, EDH, HCG, and GBF is $\frac{55}{3}$.

$FB = AB - AF = 11 - 9 = 2$.

Thus, $9x + 2(5 - x) = \frac{55}{3}$.

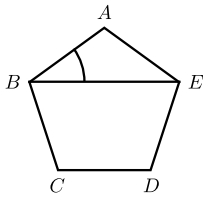
$9x + 10 - 2x = \frac{55}{3}$

Multiply by 3: $21x + 30 = 55$, so 21x=55-30=25.

Thus, $x = \frac{25}{21}$.

Area of triangle DEH is $\frac{2(5-x)}{2} = 5 - x = 5 - \frac{25}{21} = \frac{105-25}{21} = \frac{80}{21}$

4. $ABCDE$ is a regular pentagon. What is the value of $\measuredangle ABE$ in degrees (°)?

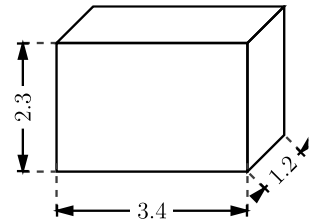


4. The sum of all angles of the pentagon is $3 \times 180 = 540°$

$\measuredangle BAE = 108°$

ΔBAE is isosceles so $\measuredangle ABE = \frac{180-108}{2} = 36°$

5. What is the total area of the 6 faces of a box with sides 1.2, 2.3, and 3.4?
Round the answer to the nearest whole number.

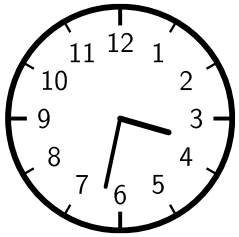


5. Surface area is:

$$2 \times (1.2 \times 2.3 + 1.2 \times 3.4 + 2.3 \times 3.4) = 2 \times (2.76 + 4.08 + 7.82) = 29.32$$

Rounding to the nearest whole number, we get that the area is approximately 29 units.

6. What is the value, in degrees, ($^\circ$), of the acute angle between the hour hand and the minute hand of a clock at the time of $3{:}\,32$ PM?



6. For each minute the minute moves $\frac{360}{60} = 6^\circ$

In 32 minutes it moves $32 \times 6 = 192^\circ$

The location of the hour hand at 3:00 is at 90°

In one hour the hour hand moves $\frac{360}{12} = 30^\circ$

In 32 minutes it moves $30 \times \frac{32}{60} = 16^\circ$

Thus the acute angle between the hands is: $192 - (90 + 16) = 192 - 106 = 86^\circ$

Worksheet #2

1. $\frac{1}{x}+\frac{1}{2x}+\frac{1}{3x}=3$. Express x as a fraction in lowest terms.

1. $6x$ is a common denominator so $\frac{6+3+2}{6x}=3$.

So, $11=6x\times 3=18x$, so $x=\frac{11}{18}$.

2. In how many ways can you pay 80 cents using any combination of 5, 10, and 25 cent coins?

2. Divide the possibilities of the sum of 80 cents into the following groups.

a) 3 0.25$, b) 2 0.25$, c) 1 0.25$, or d) 0 0.25$.

Check each of the groups for the number of options of 0.10$.

a) 0.75$ in 0.25$ coins: only 1 option for 0.10$: 0 coins of 0.10$.

b) 0.50$ in 0.25$ coins: 4 options for 0.10$: $0,1,2,$ or 3.

c) 0.25$ in 0.25$ coins: 6 options: $0,1,2,3,4,$ or 5.

d) 0.00$ in 0.25$ coins: 9 options: $0,1,2,3,4,5,6,7,$ or 8.

Total number of ways is:

$1+4+6+9=20$.

3. A triangle has sides L, M, and N, where $0<L<M<N<12$ are all whole numbers. The perimeter of the triangle is P. How many different values of P are there?

3. For any triangle with sides X, Y, and Z: $X+Y>Z$:

So, the triangle with whole numbers that satisfy the requirements, $(L=1,M,N)$ is not possible.

So the minimum possible value of P is $P=9$ of a triangle (L=2,M=3,N=4).

$P=10$ is not possible: $(L=2,M=3,N=5)$ is not a triangle because $2+3=5$. Also not acceptable are: $(L=2,M=4,N=4)$, and $(L=3,M=3,N=4)$, ($L=M$ or $M=N$ is not acceptable).

$P=11$ ok, $(2,4,5)$.

$P=12$ ok, $(3.4.5)$.

$P=13$ ok, $(3,4,6)$.

$P=14$ ok, $(3,5,6)$.

$P=15$ ok, $(3,5,7)$ and more.

And, so on, …

Max perimeter is $P=30$, $(9,10,11)$.

Summary of P values:

$\{9,11,12,13,\cdots,30\}$. So total is 21 different values of P.

4. There is a pile of 5 cards numbered $1,2,3,4,5$ on the table. Gloria takes 3 different cards at random from the pile and writes down the sum of these 3 cards. What is the probability that the sum is a multiple of 3? Express the answer as a fraction in lowest terms.

4. As shown in many workshops before, there is a formula for the number of ways to select 3 items out of 5 items:

$\frac{5!}{3!2!} = \frac{5\times4}{2\times1} = 10$.

Systematically it can easily be shown that there are 4 possible ways for the sum to be a multiple of 3:

$\{123,135,234,345\}$.

So, the probability is $\frac{4}{10} = \frac{2}{5}$.

5. x is a 2-digit positive number whose digits are two consecutive odd numbers.

y is defined to be the 2-digit positive number with the digits of x reversed.

What is the maximum possible value of $N = x + y$?

5. 79+97=176

Worksheet #3

1. Eric takes 4 times longer to paint a ceiling than to paint a wall. He charges 20% more per hour to paint a ceiling than to paint a wall. Eric painted 5 ceilings and 12 walls. His hourly charge per wall was $40 and his total earning was $2520. How many hours did he work in total?

1. Define T to be the time in hours to paint a wall.

Thus, $4T$ is the time in hours to paint a ceiling.

$40T$ is the earning in $ to paint a wall.

Thus, $4 \times 1.2 \times 40T = 192T$ is the earning in $ to paint a ceiling.

Thus, for 12 walls and 5 ceilings the earning (in $) is:

$2520 = 12 \times 40T + 5 \times 192T = (480 + 960)T = 1440T.$

Thus, $T = \frac{2520}{1440} = 1.75$ is the time in hours to paint a wall.

Total time (in hours) is then: $1.75 \times 12 + 1.75 \times 5 \times 4 = 21 + 35 = 56.$

Check the solution: $21 \times 40 + 35 \times 48 = 840 + 1680 = 2520.$

2. The following is an arithmetic sequence: $N, N + K, N + 2K, N + 3K, \cdots$. The value of the 2024-th term of the sequence is 10000, and the value of the 1000-th term is 784 . What is the value of N (the first term)?

2. The difference between the 2024-th term and the 1000-th term is: $10000 - 784 = 9216,$

Thus, $7K = \frac{9216}{1024} = 9, 10000 = N + 2023 \times 9 = N + 18207,$

So, $N = 10000 - 18207 = -8207.$

3. Express $0.5333\cdots$ as a fraction in lowest terms.

3. Multiply $x = 0.5333\cdots$ by 10. $10x = 5.333\cdots = 5 + 0.333\cdots = \frac{16}{3}.$

Thus, $x = \frac{16}{30} = \frac{8}{15}.$

4. A box contains 5 white marbles and 8 black marbles. Andrew took 2 marbles out of the box at random and placed them on the table. What is the probability that at least 1 of the marbles was white? Express the answer as a fraction in lowest terms.

4. Total number of marbles: 13. 2 out of 13: $\frac{13\times12}{2} = 13 \times 6 = 78.$

2 out of 5: $\frac{4\times5}{2} = 5 \times 2 = 10.$

1 out of 5 and 1 out of 8: $5 \times 8 = 40.$

Thus, at least one white marble: $10 + 40 = 50$. Probability $P = \frac{50}{78} = \frac{25}{39}.$

5. Of a deck of 9 cards, 4 cards are aces. Dan was dealt, (at random), 3 cards, and at least one of the cards that was dealt to him was an ace. What is the probability that he was dealt at least 2 aces? Express the answer as a fraction in lowest terms.

5. Similarly to the previous problem:

Total number of options (from below): 40+30+4=74

exactly 1 ace (1 (out 4) and 2 out of 5): $4 \times \frac{5\times4}{2} = 2 \times 5 \times 2 = 40$.

exactly 2 aces (2 (out 4) and 1 out of 5): $\frac{4\times3}{2} \times 5 = 30$.

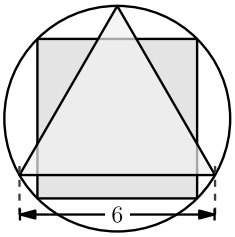
exactly 3 aces (3 (out 4) and 0 out of 5): 4.

Total number of options (from above): $40 + 30 + 4 = 74$.

At least 2 aces: $30 + 4 = 34$. Probabilty $P = \frac{34}{74} = \frac{17}{37}$.

Worksheet #4

1. An equilateral triangle and a square are inscribed in the same circle.
   The length of the side of the triangle is 6. Calculate the area of the square.



1. Let r be the radius of the circle and use the Pythagorean theorem: $r^2 = \frac{r^2}{4} + 3^2$.

Thus, $3r^2 = 36$.

So, $r = \sqrt{12} = 2\sqrt{3}$. The value of diameter of the circle, which is also the diagonal of the square, is $4\sqrt{3}$. Let x be the side of the square and use the Pythagorean theorem again: $2x^2 = 16 \times 3 = 48$.

So, the area of the square is $x^2 = 24$.

2. A box with whole number sides L, M, and N has volume $V = 3059$.
   Another box with the same volume, V, has sides $L + 4$, $M - 4$, and N.
   What is the value of $L + M + N$?

2. We know that 3059 is not divisible by 3. Divide it then by 7 to get 437. 437 has only 2 factors, namely 19 and 23. $19 + 4 = 23$, thus,

$L = 19, M = 23, N = 7, L + M + N = 19 + 23 + 7 = 49$.

3. How many pairs (N, M) are there such that $N > 0$, $M > 0$ are
   whole numbers, and $3N + 7M = 100$?

3. Find the possible values of $100 - 7 \times M$.

The list is: $\{93,86,79,72,65,58,51,44,37,30,23,16,9,2\}$.

The condition is satisfied by $4\{93,72,51,30,9\}$.

So 5 pairs satisfy the condition namely: $\{(31,1), (24,4), (17,7), (10,10), (3,13)\}$.

4. What is the largest even 9-digit number that is
   divisible by 9 and its digit sum is 63 ?

4. The sum of the digits has to be a multiple of 9 and the unit's digit has to be even.

So the largest possible of such is: 999999900

5. Three dice are rolled. What is the probability that the sum of the dice is less than 8 ? Express the answer as a fraction in lowest terms.

5. Each die has 6 options,namely $(1,2,3,4,5,6)$. So the total number of option is $6 \times 6 \times 6 = 216$.

Options on the sum:

3: $(1,1,1)$, only 1 possiblty.

4: $(1,1,2)$, 3 possibilities

5: $(1,1,3)$, 3 possibilities; $(1,2,2)$, 3 possibilities.

6: $(1,1,4)$, 3 possibilities; $(1,2,3)$, 6 possibilities; $(2,2,2)$, 1 possibility.

7:$(1,1,5)$, 3 possibilities; $(1,2,4)$, 6 possibilities; $(1,3,3)$, 3 possibilities; $(2,2,3)$, 3 possibilities.

Number of options: $1 + 3 + 6 + 10 + 15 = 35$. Thus $P = \frac{35}{216}$.

6. One of the water tanks of the Vancouver Aquarium has two taps. The first tap fills it at a rate of 15 litres per minute and the second tap at 20 litres per minute. Both taps were turned on when the tank was empty. But, the drain pipe was partly open and water was leaking out of the tank at a rate of 6 litres per minute during the first 2 hours before it was closed. The tank was finally filled up in 6 hours. What is the volume of the tank in m^3, (cubic metres)? Express the answer as decimal correct to two decimal places.

6. 6 hours is 360 minutes. Writing the equation in litres:

$$(20 + 15) \times 360 - 6 \times 120 = 60 \times (210 - 12) = 12600 - 720 = 11880.$$

Divide by 1000 to convert litres to cubic metres: 11.88.