

Workshop 2026/03/28

Introduction

Format of the competition:

3 stages

The first 2 stages are written and the students must provide, (only one), clearly legible answer in a designated area.

Stage 1 (written): Sprint. 26 questions over 3 pages, 50 minutes for the entire stage, each correct answer is worth 1 point.

First page: 9 relatively easy questions.

Second page: 9 harder questions.

Third page: 8 hardest questions.

Stage 2, (written): Target. 12 questions over 3 pages, 15 minutes per page. Each correct answer worth 2 points. For each page, the first 2 questions are easier than the last 2 question.

Maximum number of points for the first 2 stages, (combined): 50.

Stage 3, (verbal): Count Down. Top 10 students per grade participate.

Top 10 students receive medals. Top 3 students receive trophies and prizes.

Answers must be provided in one of a few prespecified format: integer, fraction in lowest term, decimal (correct to a certain number of digits), or otherwise specified format. When applicable, the units are already entered in the designated where answers should be entered.

Few important definitions and terminology, (of various type of numbers):

Whole Number, (positive). Examples 1, 4, 11

Odd number, (either positive or negative): any number that is not divisible by 2. Examples 3,-5,17

Even number, (either positive, zero, or negative): any number that is divisible by 2. Examples -10,0,32

Prime number: any whole number greater than one that is only divisible by itself and by 1. Examples 2,3,17

The K -th power of a number N : The notation is N^K . It is defined as $N \times N \times N \cdots \times N$, (K times).

Provide handout #1

Basic terminology of 2-D, (two dimensional), geometric shapes, (or objects):

A regular polygon: a 2-D shape whose boundaries are all straight lines, and all the angles between any 2 of these lines are the same.

Examples of regular polygons: equilateral triangle, square, regular pentagon, regular hexagon, regular octagon.

Some definitions of angles: **Acute**: $0 < \alpha < 90^\circ$, **Right**: $\alpha = 90^\circ$, **Obtuse**: $90^\circ < \alpha < 180^\circ$.

Sum of all the angles of a triangle: 180° .

Sum of all the angles of a polygon: any polygon with N sides can be divided by straight lines into $N - 2$ triangles. Thus, the sum of all its angles in degrees is: $(N - 2) \times 180$.

Triangles:

A triangle is a 2-D shape made up of 3 straight lines.

Right triangle: one of the angles is 90° .

Isosceles triangle: the length of two of the sides of the triangle is the same.

Equilateral triangle: the length of all the sides of the triangle is the same.

Obtuse triangle: one of the angles of the triangle is more than 90° .

Pythagorean theorem: every right triangle satisfies $a^2 + b^2 = c^2$, where a, b, c are its sides (c is the largest side of the triangle).

Some definitions of non-regular polygons with 4-sides:

Rectangle: all angles of polygon are right angles.

Parallelogram: 2 pairs of parallel sides.

Rhombus: the length of all sides is the same.

A **Circle** is defined as a round 2-D shape. The **Radius** of the circle is a straight line connecting its centre with its boundary. The **Diameter** of the circle is a straight line dividing the circle into 2 areas of equal size. Note that this line goes through the centre of the circle and its length is twice as large as the length of the radius.

The **Circumference** of the circle is the length of its rounded boundary. There is a certain constant named π that relates the circumference and the area of a circle to its radius, r . π is not a rational number, (see below for the definition of a rational number), and its value is approximately $\pi \approx 3.14 \dots$.

Circumference, $C = 2\pi r$. Area, $A = \pi r^2$.

A **Sector** of a circle is a shape surrounded by 2 radii and portion of boundary of the circle.

Definitions and properties of simple 3-D objects that have 8 **Corners** such as a **Cube** or a **Box**, and whose all angles are right angles:

The 12 **Edges**, (the lines connecting 2 adjacent corners), are straight lines.

The 6 **Faces** are the 2-D shapes enclosed in 4 edges.

If x, y, z are the edges of a box, its total **Surface area** is $S = 2(xy + xz + yz)$ and its **Volume** is given by $V = xyz$.

Provide handout #2

Fractions and decimals

A **rational number**, (or otherwise called a fraction) is a number that is either $\pm \frac{m}{n}$ where n is a, (positive), whole number and m is either zero or a, (positive), whole number.

Division of a positive whole number p by another positive whole number q , i.e. $\frac{p}{q}$, may be a whole number, or may have a **Remainder**, $0 < r < q$. Note that p is called the **Numerator**, and q is called the **Denominator**. If this division has no remainder, then the value of $\frac{p}{q}$ is called the **Quotient**.

A **Factor** of a whole number n is another whole number m that divides n .

For any **Fraction** where the numerator and the denominator have no common factor, that fraction is called: “**Fraction lowest terms.**”

Probability and statistics

A **Random** event is an event with no certainty that it will happen. For example, if tossing a fair coin there is “about” a 50% chance that the toss will be “Head” and “about” 50% chance that the toss will be “Tail”. The terminology used to describe the chances of events to occur is called the **Probability** of these events.

Thus, the probability P of any event satisfies $0 \leq P \leq 1$.

The probability of an event to occur may change if more information is given. The terminology used to define such probability is called: “**Conditional probability**”.

For example: You roll 2 dice, and it is easy to show that the probability that the event “the sum of the 2 numbers showing is less than 5”, (i.e. 2,3, or 4), is $\frac{6}{36} = \frac{1}{6}$, or to be more specific, 5 rolls, (out of total possible of 36 different rolls), satisfy the condition, namely: (1,1), (1,2), (1,3), (2,1), (2,2), (3,1). If you know that the sum is also less than 12, (the additional information), then the roll of (6,6) is excluded as a possible roll, so the total number of possible rolls is only 35, and the probability that the sum is less than 5 is now:

$$P = \frac{6}{35}$$

Provide handout #3

Workshop handouts

Handout #1

1. The sum of Abe’s and Bob’s heights is 220cm (centimetres). Abe’s height minus one third of Bob’s height is half of Abe’s height. What is Bob’s height (in cm) ?

ANSWER-7 132 (cm)

Abe’s height is A. Bob’s height is B.

$$A+B=220,$$

$$A-B/3=A/2,$$

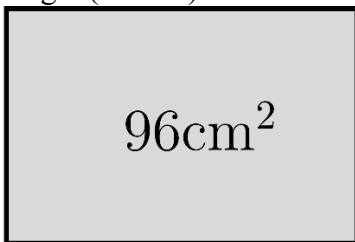
$$\text{Thus, } A/2=B/3, A=2B/3, 3A=2B$$

$$660=3A+3B=2B+3B=5B, B=660/5=132$$

$$A=220-132=88$$

$$88+132=220, 88-44=44$$

2. A rectangle has perimeter 40cm and area 96cm^2 . What is the length of the longest side of the rectangle (in cm) ?



ANSWER-7 12

Sides a.b

$$2(a + b) = 40\text{cm}$$

$$a + b = 20$$

$$ab = 96$$

$$a \times (20 - a) = 20a - a^2 = 96$$

$$\text{So, } a^2 - 20a + 96 = 0 = (a - 8) \times (a - 12)$$

$$\text{So, } a = 8 \text{ or } a = 12$$

So the longest side is 12

3. How many positive whole numbers n satisfy $n = m^k$ where m and k are also positive whole numbers, $k < 5$, and the number of digits of n is k ?

ANSWER-7 24

Suppose that the number of digits of n is 1.

So, we consider $n = m^1$. So, $n = n^1$. So, total of 9 numbers

Suppose that the number of digits of n is 2.

So, $9 < n < 100$ satisfies $n = m^2$. So, n could be 16, 25, 36, 49, 64, 81. Total 6 numbers

Suppose that the number of digits of n is 3.

So, $99 < n < 1000$ satisfies $n = m^3$. So, n could be 125, 216, 343, 512, 729. Total of 5 numbers

Suppose that the number of digits of n is 4.

So, $999 < n < 10000$ satisfies $n = m^4$. $6^4 = 1296$, 7^4 , 8^4 , 9^4 . Total of 4 numbers

No more as $k < 5$

$$9+6+5+4=24$$

4. There are 15 students in a judo class, 12 have a blue belt and 3 have a black belt. The students are split into 3 groups of 5 students to practice. Every group must have 4 blue belt students and 1 black belt student.

How many ways are there to split the class into these groups?

ANSWER-7 34650

Selection of Blue in group 1: select 4 of 12, $9 \cdot 10 \cdot 11 \cdot 12 / (1 \cdot 2 \cdot 3 \cdot 4) = 9 \cdot 5 \cdot 11 = 495$

Selection of Blue in group 2: select 4 of 8, $(5 \cdot 6 \cdot 7 \cdot 8) / (1 \cdot 2 \cdot 3 \cdot 4) = 5 \cdot 2 \cdot 7 = 70$

Total: $5 \cdot 495 \cdot 70 = 35000 - 350 = 34650$

Handout #2

1. Two buses leave the station once an hour: a red bus leaves at 47 minutes past the hour and a blue bus at N minutes past the hour. Lili arrives once daily at the station at random time and boards on the next bus that departs. On average, she boards on the red bus in 65% of the days. What is the value of N ?

ANSWER-7 8

65% of 60 minutes is 39 minutes

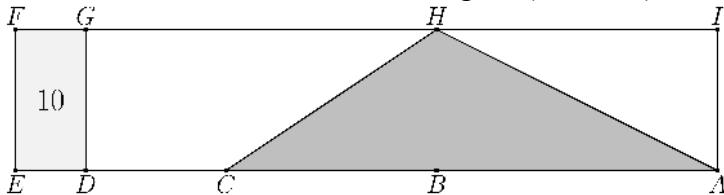
$$47 - 39 = 8$$

2. A new light bulb loses less than 0.5% of its brightness per year. That means it is at least 99.5% of its original brightness after one year, and at least 99.0025% of its original brightness after 2 years. If it still works after 8 years, what is its minimum possible brightness, as a percentage of its original brightness at that time? Round the answer to the nearest whole number.

ANSWER-7 96

After 8 years the brightness is around -96.069%

3. In the figure below $AB = 2CD$, $BC = 3DE$, $CD = 2DE$. The area of rectangle $DEFG$ is 10 cm^2 . Find the area of shaded region ($\triangle ACH$) in cm^2 .



ANSWER-7 35

Let a, b, c, d be the areas of the rectangles from right to left.

$$AB = 2CD \text{ so } a = 2c.$$

$$BC = 3DE \text{ so } b = 3d.$$

$$CD = 2DE \text{ so } c = 2d$$

$$d = 10$$

The area of the shaded region is $(a+b)/2$.

$$(a+b)/2 = (2c+3d)/2 = (4d+3d)/2 = 3.5d, \text{ so the area of the shaded region is } 35$$

4. N and K are positive whole numbers. The greatest common factor of N and K is 6. The smallest common multiple is 180. What is the smallest possible value of $N + K$?

ANSWER-7 66

$$6 = 2 \times 3$$

$$180 = 6 \times 2 \times 3 \times 5$$

$$N = 6, K = 6 \times 2 \times 3 \times 5$$

Sum of 6 and 180 is 186

$$N = 12, K = 6 \times 3 \times 5 = 90$$

Sum of 12 and 90 is 102

$$N = 18, K = 6 \times 2 \times 5 = 60$$

Sum of 18 and 60 is 78

$$6N = 30, K = 6 \times 2 \times 3 = 36$$

Sum of 30 and 36 is 66

Handout #3

1. What is the smallest whole number greater than 50 that can be expressed as the sum of two different prime numbers in exactly two different ways?

ANSWER-7 62

The number needs to be the sum of 2 odd numbers so it must be even.

$$52=5+47=11+41=23+29... \text{ NG}$$

$$54=7+47=11+43=13+41... \text{ NG}$$

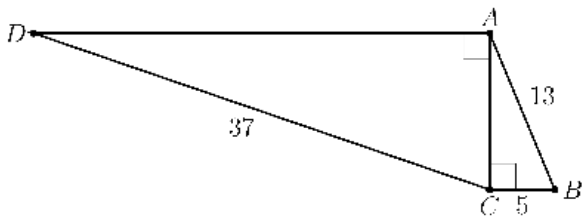
$$56=3+53=13+43=19+37... \text{ NG}$$

$$58=5+53=11+47=17+41... \text{ NG}$$

$$60=7+53=13+47=17+43... \text{ NG}$$

$$62=3+59=19+43. \quad 5, 7, 11, 13, 17, 23, 29 \text{ do not work,}$$

2. In the figure below, $AB = 13$, $BC = 5$, AC perpendicular to both BC and AD , and $CD = 37$. What is the area of the polygon $ABCD$?



ANSWER-7 240

$$AC = \sqrt{13^2 - 5^2} = \sqrt{144} = 12$$

$$AD = \sqrt{37^2 - 144} = \sqrt{1369 - 144} = \sqrt{1225} = 35$$

Thus, the area of ABCD is $12 \cdot 5/2 + 12 \cdot 35/2 = 12 \cdot 20 = 240$

3. 1000 runners competed in a very long race and 82% did not finish. 30% of the runners aged 20 or older finished the race and they made up 75% of all the people who finished. How many participants were less than 20 years old?

ANSWER-7 550

18% of 1000 is 180. So, 180 finished. People aged 20 or older who finished is $180 \times 75\% = 45 \times 3 = 135$. Total of 20 or older participants are $135/30\% = 45 \times 10 = 450$. Total 20 or younger participants is $1000 - 450 = 550$.

4. How many non congruent triangles with whole number sides have perimeter of 17?

ANSWER-7 8

For every triangle the sum of the lengths of 2 of its sides is always larger than the length of the third side.

The triangle that satisfy the conditions are:

The longest side must be 8 or less:

(8,8,1), (8,7,2), (8,6,3), (8,5,4),

(7,7,3), (7,6,4), (7,5,5),

(6,6,5)

The largest side cannot be less than 6.

Total of number of triangles: 8

Handout #4

1. A bag contains 8 red balls, 6 blue balls and 6 green balls. Three balls are selected at random, without replacement. If the first ball was red, what is the probability that all of the selected balls will be different colours? Express the answer as a fraction in lowest term.

ANSWER-7 $\frac{4}{19}$

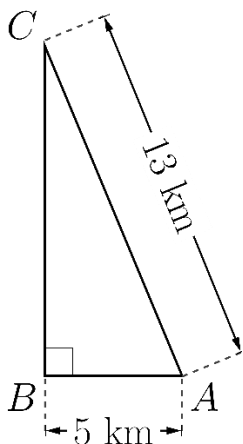
The first ball, red, has already been selected so, in the bag there are now 7R, 6B, and 6G, 19 balls in total to select 2 remaining balls.

Number of ways to select 2 out of 19 is: $18 \cdot 17 / 2 = 171$

One blue and one green: $6 \cdot 6 = 36$

$36 / 171 = 4 / 19$

2. $\triangle ABC$ is a right triangle ($\angle ABC = 90^\circ$). $AB = 5\text{ km}$, $AC = 13\text{ km}$. Ann and Bob start travelling from point A at the same time. Ann travels from point A to point C along the line AC at speed of $4 \frac{\text{km}}{\text{h}}$, (kilometres per hour). Bob travels from point A to point B along the line AB at speed of $6 \frac{\text{km}}{\text{h}}$ and then continues, without stopping, to point C along line BC at a constant speed V (in $\frac{\text{km}}{\text{h}}$). Ann and Bob arrive at point C at the same time. What is the value of V in $\frac{\text{km}}{\text{h}}$? Express the answer as a fraction in lowest terms.



ANSWER-7 $\frac{144}{29}$

Using Pythagorean theorem we get: $BC = \sqrt{13^2 - 5^2} = \sqrt{144} = 12$

Travel time in hours for Ann: $13/4$.

Travel time in hours for Bob from A to B: $5/6$.

Travel time in hours for Bob from B to C: $13/4 - 5/6 = 39/12 - 10/12 = 29/12$.

Speed of Bob between B and C: $12/(29/12)=144/29$

3. Taylor receives an award for outstanding achievements in mathematics. She deposits 40% of the award in her bank account, and uses $\frac{7}{12}$ of what remaining to cover her travel expenses. She donates the rest of the money to charity, giving $\frac{2}{3}$ of that to United Way and the remaining \$750 to the Red Cross. How big was the initial award (in dollars)?

ANSWER-7 9000

Let A to be the total reward.

$$\text{So, } A \times \frac{6}{10} \times \frac{5}{12} \times \frac{1}{3} = 750$$

$$\text{So, } A \times \frac{1}{12} = 750$$

$$A = 750 \times 12 = 9000$$

4. The total value of all the coins in a bag is \$85.00. There are equal numbers of each of the following coin values: \$0.05, \$0.10, \$0.25, \$1.00, \$2.00. There are no coins of any other values in the bag.

How many coins are there in the bag?

ANSWER-7 125

$$0.05+0.10+0.25+1.00+2.00=3.40$$

$$\$85/\$3.40=25$$

5 different denominations so $5 \times 25 = 125$

Handout #5

1. Bob, lives in Burnaby and invites some of his friends to a party. 17 of his friends are from Vancouver and 21 are from Richmond. Each guest shakes hands with all of the other people from the same city as them (once), and each guest shakes hands with Bob (once). How many handshakes are there in total?

ANSWER-7 384

$$9 \times 4 + 21 \times 10 + 30 = 36 + 210 + 30 = 276$$

$$13 \times 6 + 21 \times 10 + 34 = 78 + 210 + 34 = 322$$

$$17 \times 16/2 + 21 \times 20/2 + (17+21) = 136 + 210 + 38 = 384$$

2. A 2-digit whole number is randomly selected. If the digit sum of this number is 9, what is the probability that the number is less than 80? Express the answer as a fraction in lowest terms.

ANSWER-7 $\frac{7}{9}$

The following 9 numbers are the numbers with digit sum of 9: 18, 27, 36, 45, 54, 63, 72, 81, and 90.

Two of this are higher than 80 so the probability is $7/9$

3. Given that $p_1 \times p_2 \times \dots \times p_N = 2025$ where p_1, p_2, \dots, p_N are all primes. What is the value of $p_1 + p_2 + \dots + p_N$?

ANSWER-7 22

$$2025 = 25 \times 81 = 5 \times 5 \times 3 \times 3 \times 3 \times 3$$

$$5 + 5 + 3 + 3 + 3 + 3 = 22$$

4. What is the sum of the 3 smallest whole numbers that have exactly 18 factors? Hints: 1) The value of each of these 3 numbers is less than 300. 2) The 3 smallest whole numbers that have exactly 2 factors are 2,3,5. So their sum is 10.

ANSWER-7 720

Numbers that have 18 factors:

$$p^{17}$$

$$2^{17} > 300 \text{ NG}$$

$$p \times q^8$$

Number of factors is $A(1 + 1) \times (1 + 8) = 18$

$$3 \times 2^8 > 300 \text{ NG}$$

$$p^2 \times q^5$$

$$2^2 \times 3^5 > 300 \text{ NG}$$

$$3^2 \times 2^5 = 288$$

$$5^2 \times 2^5 > 300 \text{ NG}$$

$$p \times q^2 \times r^2$$

$$2 \times 3^2 \times 5^2 > 300 \text{ NG}$$

$$3 \times 2^2 \times 5^2 = 300 \text{ OK}$$

$$3 \times 2^2 \times 7^2 > 300 \text{ NG}$$

$$5 \times 2^2 \times 3^2 = 180 \text{ OK}$$

$$5 \times 2^2 \times 7^2 > 300 \text{ NG}$$

$$7 \times 2^2 \times 3^2 = 252 \text{ OK}$$

$$7 \times 2^2 \times 5^2 > 300 \text{ NG}$$

$$11 \times 2^2 \times 3^2 > 300 \text{ NG}$$

Candidates under 500: 180(1), 252(2), 288(3), 300(4), 396(5)

$$180+252+288=720$$